Summer 2014
Review Day 10

1 Conservative Fields

1.1 Verify that $F = \nabla\phi$ and evaluate the line integral of F over the given path.

(a) $F = \langle 3, 6y\rangle$, $\phi(x, y) = 3x + 3y^2$; $c(t) = (t, 2t^{-1})$ **for** $1 \leq t \leq 4$.

(b) $F = \langle xy^2, x^2y\rangle$, $\phi(x, y) = \frac{1}{2}x^2y^2$; **upper half of the unit circle centered at the origin oriented counterclockwise.**

(c) $F = \langle ye^z, xe^z, xye^z\rangle$, $\phi(x, y, z) = xye^z$; $c(t) = (t^2, t^3, t-1)$ **for** $1 \leq t \leq 2$.

1.2 Determine whether the vector field is conservative and if so, find a potential function.

(a) $F = \langle z, 1, x\rangle$

(b) $F = \langle 0, x, y\rangle$

(c) $F = \langle y^2, 2xy + e^z, ye^z\rangle$

(d) $F = \langle y, z, z^3\rangle$

(e) $F = \langle \cos(xz), \sin(yz), xy\sin(z)\rangle$

(f) $F = \langle \cos(z), 2y, -x\sin(z)\rangle$

2 Green's Theorem

2.1 Verify Green's Theorem for the line integral $\oint_C xydx + ydy$ where C is the unit circle, oriented counterclockwise.

2.2 Use Green's Theorem to evaluate the line integral. Orient the curve counterclockwise unless otherwise indicated

(a) $\oint_C y^2dx + x^2dy$, **where** C **i the boundary of the unit square** $0 \leq x, y \leq 1$.

(b) $\oint_C x^2ydx$, **where** C **is the unit circle centered at the origin.**

(c) $\oint F \cdot ds$, **where** $F = \langle x^2, x^2\rangle$ **and** C **consists of the arcs** $y = x^2$ **and** $y = x$ **for** $0 \leq x \leq 1$

2.3 Use Green's Theorem to calculate the area of the given region

(a) The circle of radius 3 centered at the origin

(b) The region between the x-axis and the cycloid parametrized by $c(t) = (t - \sin t, 1 - \cos t)$ **for** $0 \leq t \leq 2\pi$**.**