Summer 2014
Review Day 4

Gradients and Chain Rule

1. Let $f(x, y)$ be a multivariable function.

(a) Write the formula for the gradient, ∇f.

(b) Write the formula for the directional derivative, $D_v f$, where $v = \langle h, k\rangle$.

(c) Let $c(t)$ be a path. Write the formula for chain rules for paths, $\frac{d}{dt} f(c(t))$

2. Let $f(x, y) = xy^2$

(a) Compute ∇f.

(b) Compute $D_v f$ in the direction $v = \langle 1, 2\rangle$.

(c) Let $c(t) = (t^2, t^3)$. Compute $\frac{d}{dt} f(c(t))$.

3. Gradients of functions with 50% more variables.

(a) Let $f(x, y, z) = xyz^{-1}$. Compute ∇f.

(b) Compute $D_v f$ in the direction $v = \langle 1, 2, 3\rangle$

(c) Let $c(t) = (e^t, t, t^2)$. Compute $\frac{d}{dt} f(c(t))$.

4. Write an equation of the tangent plane to $x^2 + 3y^2 + 4z^2 = 20$ at the point $P = (2, 2, 1)$.

(a)While you are at it, write the general equation for the tangent plane to the level surface $F(x, y, z) = k$ at the point $P = (a, b, c)$

5. Let $f(x, y) = (x(s, t), y(s, t))$

(a) Write the formula for $\frac{\delta f}{\delta s}$

(b) Compute $\frac{\delta f}{\delta s}$ for $f(x, y) = (ts^2, \cos(t)\ln(s))$

Optimization

1. Give the definition of a critical point for a multivariable function

(a) Find all critical points of the function $x^2(y-1)^2$**.**

(b) Find all the critical points of the function $x^2 \ln(y+1)$

2. Second Derivative Test

(a) What is the second derivative test?

(b) Write the formula for the discriminant D**.**

(c) What does it mean if $D > 0$

(d) What does it mean if $D < 0$

(e) What does it mean if $D = 0$

(f) What does it mean if $D > 0$ **and** $f_{xx}(a,b) > 0$

(g) What does it mean if $D > 0$ **and** $f_{xx}(a,b) < 0$

(h) What does it mean if $D > 0$ **and** $f_{xx}(a,b) = 0$

3. Find all critical points and determine whether they are local minima or maxima

(a) $f(x,y) = x^4 + y^4 - 4xy$

(b) $f(x,y) = x - y^2 - \ln(x+y)$

(c) $f(x,y) = xye^{-x^2-y^2}$

4. Show that the rectangular box (including a top and bottom) with fixed volume V with the smallest possible surface area is a cube.

Hint: Write the formula for the volume the box and surface area.