**Summer 2014**
**Review Day 6**

## Double Integrals

**1. Evaluate**

**(a)** $\int_1^3 \int_0^2 x^3 y \ dydx$

**(b)** $\int_0^2 \int_1^3 x^3 y \ dydx$

**(c)** $\int_0^1 \int_2^3 \sqrt{x+4y} \ dxdy$

**(d)** $\int_0^{\pi/4} \int_0^{\pi/2} \cos(2x+y) \ dydx$

**(e)** $\int_0^1 \int_2^3 \frac{1}{(x+4y)^3} \ dxdy$

**(f)** $\int_1^2 \int_1^2 \ln(xy) \ dydx$

**2. Evaluate the double integral of the function over the rectangle**

**(a)** $\int\int_R (2x+6y) \ dA, \ R = [0,3] \times [0,2]$

**(b)** $\int\int_R \cos(x)\sin(2y) \ dA, \ R = [0,\pi/2] \times [0,\pi/2]$

**(c)** $\int\int_R \frac{y}{x+1} \ dA, \ R = [0,2] \times [0,4]$

**3. Sketch the Region R. Evaluate the double integral of the function over the Region.**

**(a)** $\int\int_R x^3 \ dA, \ R = [0,2] \times [x^2,4]$

**(b)** $\int\int_R xy \ dA, \ R = [1,3] \times [x,2x+1]$

**(c)** $\int\int_R x \ dA, \ R = [0,1] \times [1,e^x]$

**(d)** $\int\int_R \frac{y}{x} \ dA, \ R = [1,e^y] \times [0,1]$

**(e)** $\int\int_R (x+y)^{-1} \ dA, \ R = [0,y] \times [1,e]$

**4. For each double integral, sketch the region. Then change the order of integration and evaluate.**

**(a)** $\int_0^1 \int_y^1 \frac{\sin(x)}{x}\ dxdy$

**(b)** $\int_0^1 \int_x^1 xe^{y^3}\ dydx$

**(c)** $\int_0^9 \int_0^{\sqrt{y}} \frac{x^3\ dxdy}{\sqrt{3x^2+y}}$

**5. Compute the double integral of** $f(x, y) = ye^x$ **over the triangle with corners at** $(0, 0), (1, 4), (3, 4)$

**6. Find the average value of the function** $f(x, y)$ **over the square** $[0, 1] \times [0, 1]$ **where:**

**(a)** $f(x, y) = e^{x+y}$

**(b)**$f(x, y) = \frac{1}{x^2-1}$

**7. Find the volume of the region bounded by the paraboloids** $z = x^2 + y^2$ **and** $z = 8 - x^2 - y^2$**.**

**(Hint: Recall how we solved for the area between two curves in Calc 1. Finding volume is the exact same process, except with one more dimension.**

**8. Set up a double or triple integral that will give you the volume of a unit sphere.**