Summer 2014
Review Day 5

Lagrange Multipliers

1. Use Lagrange multipliers to find the extreme values of the function $f(x, y) = 2x + 4y$ subject to the constraint $g(x, y) = x^2 + y^2 - 5 = 0$.

(a) Show that the Lagrange equations $\nabla f = \lambda \nabla g$ gives $\lambda x = 1$ and $\lambda y = 2$.

(b) Show that these equations imply $\lambda \neq 0$ and $y = 2x$.

(c) Use the constraint equation to determine the possible critical points (x, y).

(d) Evaluate $f(x, y)$ at the critical points and determine the minimum and maximum values.

2. Apply the method of Lagrange multipliers to the function $f(x, y) = (x^2 + 1)y$ subject to the constraint $x^2 + y^2 = 5$.

Hint: First show that $y \neq 0$, then treat the case $x = 0$ and $x \neq 0$ separately

3. Find the minimum and maximum values of the functions subject to the given constraint.

(a) $f(x, y) = x^2 + y^2$, $2x + 3y = 6$.

(b) $f(x, y) = xy$, $4x^2 + 9y^2 = 32$.

(c) $f(x, y) = x^2 + y^2, x^4 + y^4 = 2$.

(d) $f(x, y, z) = 3x + 2y + 4z$, $x^2 + 2y^2 + 6z^2 = 1$

4. Find the rectangular box of maximum volume if the sum of the lengths of the edges is 300 cm.

5. Find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to two constraints, $x + 2y + z = 3$ and $x - y = 4$.

Review

Do the following WITHOUT looking at notes or a textbook. Try to answer at least fifty percent of the questions.

1. Evaluate the limit or prove that it does not exist

$$\lim_{(x,y)\to(1,-3)} (xy + y^2)$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3y^2 + x^2y^3}{x^4 + y^4}$$

2. Compute f_x and f_y

(a) $f(x, y) = 2x + y^2$

(b) $f(x, y) = \sin(xy)e^{-x-y}$

3. Compute f_{xxyz} for $f(x, y, z) = y \sin(x + z)$

3. Find an equation of the tangent to the graph of $f(x, y) = xy^2 - xy + 3x^3y$ at $P = (1, 3)$.

4. Suppose that the plane $z = 2x - y - 3$ is tangent to the graph of $z = f(x, y)$ at $P = (2, 4)$

(a) Determine $f(2, 4), f_x(2.4),$ and $f_y(2.4)$.

(b) Approximate $f(2.2, 3.9)$.

5. Compute $\frac{d}{dt}f(c(t))$ where $f(x, y) = x + e^y$, $c(t) = (3t - 1, t^2)$ at $t = 2$.

6. Compute the directional derivative at P in the direction of v, $f(x, y) = x^3y^4$, $P = (3, -1)$, $v = 2i + j$.

7. Let $f(x, y) = (x - y)e^x$. Use the chain rule to calculate $\frac{\delta f}{\delta u}$ where $x = u - v$ and $y = u + v$.

8. Find all local min, max and saddle points of $f(x, y) = x^4 - 2x^2 + y^2 - 6y$

9.Use Lagrange multipliers to find the minimum and maximum value of $f(x, y) = 3x - 2y$ on the circle $x^2 + y^2 = 4$.

10. Use Lagrange multipliers to find the dimensions of a cylindrical can of fixed volume V with minimal surface area (including the top and bottom of the can)