Vector Fields

1. Compute and sketch the vector assigned to the given points by the given vector field.

(a) $P = (1, 2)$**,** $Q = (-1, -1)$**, by the vector field** $F = \langle x^2, x \rangle$.

(b) $P = (0, 1, 1)$**,** $Q = (2, 1, 0)$ **by the vector field** $F = \langle xy, z^2, x \rangle$**.**

2. Sketch the following planar vector fields by drawing the vectors attached to points with integer coordinates in the rectangle $-2 \leq x, y \leq 2$**. Instead of drawing the vectors with their true lengths, scale them if necessary to avoid overlap.**

(a) $F = \langle 1, 0 \rangle$

(b) $F = \langle 0, x \rangle$

(c) $F = \langle 1, 1 \rangle$

(d) $F = \langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \rangle$

(e)$F = \langle \frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2} \rangle$

Line Integrals

1. Let $f(x, y, z) = x + yz$ **and let** C **be the line segment from** $P = (0, 0, 0)$ **to** $(6, 2, 2)$**.**

(a) Calculate $f(c(t))$ **and** $ds = ||c'(t)||dt$ **for the parametrization** $c(t) = (6t, 2t, 2t)$ **for** $0 \leq t \leq 1$**.**

(b) Evaluate $\int_C f(x, y, z)$**.**

2. Let $F = \langle y^2, x^2 \rangle$ **and let** C **be the** $y = x^{-1}$ **for** $1 \leq x \leq 2$**, oriented from left to right.**

(a) Calculate $F(c(t))$ **and** $ds = c'(t)dt$ **for the parametrization** $c(t) = (t, t^{-1})$**.**

(b) Calculate the dot product $F(c(t))\dot{c}'(t)dt$ **and evaluate** $\int_C F\dot{d}s$**.**

3. Calculate the integral of the given scalar function or vector field over the curve $c(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq \pi$.

(a) $f(x, y, z) = x^2 + y^2 + z^2$

(b) $F = \langle x, y, z \rangle$

4. Calculate the total mass of a circular piece of wire of radius 4 cm centered at the origin whose mass density is $\rho(x, y) = x^2$g/cm.

5. Compute the line integral of the scalar function over the curve.

(a)$f(x, y, z) = z^2$**, for the line** $c(t) = (2 + t, 2 - t, 2t)$ **for** $-2 \leq t \leq 1$**.**

(b)$f(x, y, z) = \sqrt{1 + 9xy}$**, for the line** $y = x^3$ **for** $0 \leq x \leq 1$**.**

(c)$f(x, y, z) = xe^{z^2}$**, piecewise linear path from** $(0, 0, 1)$ **to** $(0, 2, 0)$ **to** $(1, 1, 1)$

(d)$f(x, y, z) = 2x^2 + 8z$, **for the line** $c(t) = (e^t, t^2, t)$, **for** $0 \leq t \leq 1$.

6. Compute the line integral of the vector field over the oriented curve

(a)$F = \langle x^2, xy \rangle$**, line segment from** $(0, 0)$ **to** $(2, 2)$**.**

(b)$F = \langle xy, x + y \rangle$**, circle** $x^2 + y^2 = 9$ **oriented clockwise**

(c)$F = \langle xy, x + y \rangle$**,** $c(t) = (t + t^2, \frac{1}{3}t^3, 2 + t)$ **for** $0 \leq t \leq 2$**.**

(d)$F = \langle 3zy^{-1}, 4x, -y \rangle$, $c(t) = (e^t, e^t, t)$ **for** $-1 \leq t \leq 1$**.**