

Problem 1. A fair die is tossed independently eight times.

- (a) Among 8 tosses, what is the probability there are at least two “6” appear?
- (b) What is the probability that the first “6” occurs on the 4th toss?
- (c) What is the probability that the second “6” occurs on the 4th toss?

Problem 2. Suppose Z follow standard normal distribution

- (a) $P(Z < 0.6)$
- (b) $P(Z > 0.6)$
- (c) $P(|Z| < 0.6)$
- (d) $P(|Z| > 0.6)$

Find the z values satisfying

- (e) $P(Z < z) = 0.6$
- (f) $P(Z > z) = 0.6$
- (g) $P(|Z| < z) = 0.6$

Problem 3. If random variable X satisfies a normal distribution with mean 60 and standard deviation 20

- (a) Determine $P(X > 75)$
- (b) Determine $P(|X - 50| < 20)$
- (c) Find c such that $P(X > c) = 0.80$
- (d) Find c such that $P(|X - 60| < c) = 0.28$

Problem 4. A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the ounces of the fill are normally distributed with the standard deviation of 0.3 ounces, find the mean μ such that 8-ounce cups will overflow no more than 2.5% of the time.

Problem 5. Let X be the number of customers login on a website per minute. Assume that X has a Poisson distribution with mean 4 logins per minutes.

- (a) What is the probability that there is at least one login on this website in the next minute?
- (b) What is the expected hourly login?

Let W denote the minutes until the first login on the website.

- (c) What is the probability density function of W ?
- (d) Calculate $P(W > 0.5)$

Problem 6. Let X have the p.d.f. $f(x) = x$, $0 < x < \sqrt{2}$

- (a) Find the mean and variance of X
- (b) Suppose X_1, X_2, X_3 are a random sample of three from the distribution of X .
Let $T = X_1 + X_2 + X_3$, calculate the mean and variance of T .
- (c) Let $Y = 3X + 2$, find its distribution function $G(y) = P(Y < y)$ and its p.d.f. $g(y)$.

Problem 7. Average weekly expenditure from a population in a given region is \$ 19.50, and the population standard deviation is \$5.3. If the weekly expenditure is normally distributed

- (a) Find the mean and standard deviation of the sample mean given a random sample of size 100.
- (b) What is the probability that a random sample of size 100 will yield a sample mean greater than \$20?

Answer:

1. (a) 0.395 (b) 0.096 (c) 0.058

2. (a) 0.7257 (b) 0.2743 (c) 0.4514 (d) 0.5486 (e) 0.25 (f) -0.25 (g) 0.84

3. (a) 0.2266 (b) 0.6247 (c) 43.2 (d) 7.2

4. $\mu = 7.412$

5. (a) 0.982 (b) 240

(c) $W \sim \text{Gamma}(\alpha = 1, \beta = \frac{1}{4})$, then p.d.f. $f(w) = 4e^{-4w}, w > 0$ (d) 0.1353

6. (a) $E(X) = \frac{2\sqrt{2}}{3}, \text{Var}(X) = \frac{1}{9}$ (b) $E(T) = 2\sqrt{2}, \text{Var}(T) = \frac{1}{3}$

(c) c.d.f of Y : $G(y) = \begin{cases} 0, & y < 2 \\ (y-2)^2 / 18, & 2 \leq y < 2 + 3\sqrt{2} \\ 1, & y \geq 2 + 3\sqrt{2} \end{cases}$

(d) p.d.f. of Y : $g(y) = \begin{cases} (y-2)/9, & 2 \leq y < 2 + 3\sqrt{2} \\ 0, & \text{otherwise} \end{cases}$

7. (a) $E(\bar{X}) = 19.5, SD(\bar{X}) = 0.53$ (b) 0.1736