STAT 381 - 13605 **Quiz 10** **2006-11-29**

Solution

1. **A random sample of n=16 observations from a normal distribution $N(\mu, \sigma^2)$ leads to sample mean $\bar{x} = 70$, sample variance $s^2 = 576$.**
   (a).[2pt] Determine a 90% confidence interval for population mean μ.

   $n = 16, d.f. = n - 1 = 15, \alpha/2 = 0.05, t_{(0.05,15)} = 1.753$

   C.I. for **μ:** $\bar{x} \pm t_{(0.05,15)} \frac{s}{\sqrt{n}} = 70 \pm (1.753 \cdot 6) = 70 \pm 10.518$

   (b).[3pt] Find a 90% confidence interval for population variance σ^2.

   $d.f. = 15, \alpha/2 = 0.05, \chi^2_{(0.05,15)} = 24.996, \chi^2_{(0.95,15)} = 7.261$

   C.I. for σ^2 :

   $$\left[\frac{(n-1)s^2}{\chi^2_{(0.05,15)}}, \frac{(n-1)s^2}{\chi^2_{(0.95,15)}}\right] = \left[\frac{15 \cdot 576}{24.996}, \frac{15 \cdot 576}{7.261}\right] = [345.65, 1189.92]$$

2. **[3pt] The makers of a medicated facial skin cream are interested in determining the percentage of people in a given age group who may benefit from the ointment. A random sample of 68 people results in 42 successful treatments. Give a 99% confidence interval for the proportion of people in the given age group who may be successfully treated with the facial cream.**

   y=42, n=68, $\hat{p} = \frac{y}{n} = \frac{42}{68} = .6176$, $\alpha = 0.01, \alpha/2 = 0.005, z_{0.005} = 2.576$

   C.I. for p:

   $$\hat{p} \pm z_{0.005}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.6176 \pm 2.576 \cdot \sqrt{\frac{0.6176 \cdot (1 - 0.6176)}{68}} = 0.6176 \pm 0.1518$$

3. **[2pt] Let Y be Binomial(*n*, *p*). Find the sample size *n* such that we are 95% confident that the estimate $\hat{p} = y/n$ is within 0.2 units of *p* given that *p* is unknown.**

   Margin of error B = 0.2, i.e. $z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{p(1-p)}{n}} = 0.2$

   $(1-\alpha) = 0.95, \alpha/2 = 0.025, z_{0.025} = 1.96$

   Then $n = \frac{z^2_{\alpha/2} \cdot p(1-p)}{B^2} \le \frac{z^2_{\alpha/2}}{4B^2} = \frac{1.96^2}{4(0.2)^2} = 24.01 \quad \nearrow 25$