1. [6pt] Suppose the probability of producing a defective item is 0.05.

a). What is the probability that exactly two is defective out of 10 items that are selected independently?

Let $X$ be the number of defective ones in the 10 items, then $X \sim B(10, 0.05)$

$\{X=2\} = \{ \text{exactly two is defective out of 10 items} \}$

Use the Binomial Table:

$P(X = 2) = \binom{10}{2} 0.05^2 (1 - 0.05)^8 = 0.0746$

Or directly $P(X = 2) = \binom{10}{2} 0.05^2 (1 - 0.05)^8 = 0.0746$

b). How many items do we have to produce so that the probability of producing at least one defective item is greater than 90%?

$P(X \geq 1) \geq 0.90$, then $1 - P(X=0) \geq 0.90$

$P(X=0) = 0.95^n$, then $1 - 0.95^n \geq 0.90$ i.e. $0.95^n \leq 0.10$

$n \geq \log(0.10)/\log(0.95)$, so $n \geq 44.89 \Rightarrow n = 45$

2. [4pt] The daily number of plant shutdown follows a Poisson distribution with mean 2.

a) What is the probability that there is at least one shutdown in a day?

Let $Y$ be the daily number of plant shutdown, then $Y \sim P(2)$

$\{Y \geq 1\} = \{ \text{at least one shutdown in a day} \}$

Use the Poisson Table: $P(Y \geq 1) = 1 - F(0) = 1 - 0.135 = 0.865$

Or $P(Y \geq 1) = 1 - P(X = 0) = 1 - \frac{2^0 e^{-2}}{0!} = 1 - e^{-2} = 1 - 0.1353 = 0.8647$

b) Assume that the company losses $10000 on each shutdown. Calculate the expected daily loss.

Let $Z$ be the daily loss of the company, $Z = 10000*Y$

Since $Y \sim P(2)$, then $E(Y) = 2$.

Hence the expected daily loss $E(Z) = 10000*E(Y) = 20000$. 