STAT 381 - 13605 Quiz 6 2006-10-25

Solution

1. **[6pt] Let X have the probability density function**

$$f(x) = \begin{cases} 3x^2, & 0 \le x < 1 \\ 0, & otherwise \end{cases}$$

a) **Find the mean of the random variable X.**

$$E(X) = \int_{-\infty}^{+\infty} xf(x)dx = \int_0^1 x(3x^2)dx = \left[\frac{3}{4}x^4\right]_0^1 = \frac{3}{4}$$

b) **Obtain the cumulative distribution function.**

If x<0, then F(x) = P(X≤x) =0

If x≥1, then F(x) = P(X≤x) = 1

If 0≤x<1, then $F(x) = P(X \le x) = \int_{-\infty}^{x} f(w)dw = \int_0^x 3w^2 dw = [w^3]_0^x = x^3$

Hence the cumulative function of X: $F(x) = \begin{cases} 0, & x < 0 \\ x^3, & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$

c) **Compute P(0.5 < X < 1.2)**

$$P(0.5 < X < 1.2) = F(1.2) - F(0.5) = 1 - (0.5)^3 = 1 - 0.125 = 0.875$$

2. **[4pt] Let Y be N(10, 4). Determine the probabilities:**

a) **P(11 < Y < 14)**

Y~ N (10, 4), then μ=10, σ = 2 (or $\sigma^2 = 4$)

$$P(11 < Y < 14) = P\left(\frac{11-\mu}{\sigma} < Z < \frac{14-\mu}{\sigma}\right) = P\left(\frac{11-10}{2} < Z < \frac{14-10}{2}\right)$$

$$= P(0.5 < Z < 2) = \phi(2) - \phi(0.5) = 0.9772 - 0.6915 = 0.2857$$

b) **P (| Y - 10| ≤2)**

$$P(|Y-10| \le 2) = P(10-2 \le Y \le 10+2) = P(8 \le Y \le 12)$$

$$= P\left(\frac{8-\mu}{\sigma} \le Z \le \frac{12-\mu}{\sigma}\right) = P\left(\frac{8-10}{2} \le Z \le \frac{12-10}{2}\right)$$

$$= P(-1 \le Z \le 1) = \phi(1) - \phi(-1) = 0.8413 - (1 - 0.8413) = 0.6826$$