EXAMPLE 1: A Measurement Systems Capability Study

Statistically designed experiments are frequently used to investigate the sources of ability that affect a system. A common industrial application is to use a designed experiment to study the components of variability in a measurement system. These studies are often called gauge capability studies or gauge repeatability and reproducibility (R&R) studies because these are the components of variability that are of interest (for more discussion of gauge R&R studies.

A typical gauge R&R experiment is shown in Table 1. An instrument or gauge is used to measure a critical dimension on a part. Twenty parts have been selected from the production process, and three randomly selected operators measure each part twice with this gauge. The order in which the measurements are made is completely randomized, so this is a two-factor factorial experiment with design factor parts and operators, with two replications. Both parts and operators are random factors.

Table 1. The Measurement Systems Capability Experiment

Part No.	Operator 1		Operator 2		Operator 3	
1	21	20	20	20	19	21
2	24	23	24	24	23	24
3	20	21	19	21	20	22
4	27	27	28	26	27	28
5	19	18	19	18	18	21
6	23	21	24	21	23	22
7	22	21	22	24	22	20
8	19	17	18	20	19	18
9	24	23	25	23	24	24
10	25	23	26	25	24	25
11	21	20	20	20	21	20
12	18	19	17	19	18	19
13	23	25	25	25	25	25
14	24	24	23	25	24	25
15	29	30	30	28	31	30
16	26	26	25	26	25	27
17	20	20	19	20	20	20
18	19	21	19	19	21	23
19	25	26	25	24	25	25
20	19	19	18	17	19	17

```
/*Full model: operator (random), part (random) in Example 1*/

proc glm data = capacity;
     class part operator;
     model measure = part operator part*operator;
     random part operator;
     test h=part e=part*operator;
     test h=operator e=part*operator;
     run;
```

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	59	1215.091667	20.594774	20.77	<.0001
Error	60	59.500000	0.991667		
Corrected Total	119	1274.591667			

R-Square	Coeff Var	Root MSE	measure Mean
0.953318	4.447300	0.995825	22.39167

Source	DF	Type I SS	Mean Square	F Value	Pr > F
part	19	1185.425000	62.390789	62.92	<.0001
operator	2	2.616667	1.308333	1.32	0.2750
part*operator	38	27.050000	0.711842	0.72	0.8614

Source	Type III Expected Mean Square
part	Var(Error) + 6 Var(part) + Q(part*operator)
operator	Var(Error) + 40 Var(operator) + Q(part*operator)
part*operator	Var(Error) + Q(part*operator)

Tests of Hypotheses Using the Type III MS for part*operator as an Error Term					
Source	DF	Type III SS	Mean Square	F Value	Pr > F
part	19	1185.425000	62.390789	87.65	<.0001
operator	2	2.616667	1.308333	1.84	0.1730

```
/*Reduced model: operator (random), part (random) in Example 1*/

proc glm data = capacity;
     class part operator;
     model measure = part operator;
     random part operator;
     run;
```

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	21	1188.041667	56.573413	64.06	<.0001
Error	98	86.550000	0.883163		
Corrected Total	119	1274.591667			

R-Square	Coeff Var	Root MSE	measure Mean
0.932096	4.196953	0.939768	22.39167

Source	DF	Type I SS	Mean Square	F Value	Pr > F
part	19	1185.425000	62.390789	70.64	<.0001
operator	2	2.616667	1.308333	1.48	0.2324

Source	Type III Expected Mean Square
part	Var(Error) + 6 Var(part)
operator	Var(Error) + 40 Var(operator)

EXAMPLE 2:

Reconsider the gauge experiment in Example 1. Suppose now that **only three operators use this gauge**, so the operators are a fixed factor. However, the parts are chosen at random, the experiment now involves a mixed model

```
/*Mixed model:  operator (fixed) and part (random) in Example 2*/
proc glm data = capacity;
     class part operator;
     model measure = part operator part*operator;
     random part;
     test h=operator e=part*operator;
     run;
```

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	59	1215.091667	20.594774	20.77	<.0001
Error	60	59.500000	0.991667		
Corrected Total	119	1274.591667			

R-Square	Coeff Var	Root MSE	measure Mean
0.953318	4.447300	0.995825	22.39167

Source	DF	Type I SS	Mean Square	F Value	Pr > F
part	19	1185.425000	62.390789	62.92	<.0001
operator	2	2.616667	1.308333	1.32	0.2750
part*operator	38	27.050000	0.711842	0.72	0.8614

Source	Type III Expected Mean Square
part	Var(Error) + 6 Var(part) + Q(part*operator)
operator	Var(Error) + Q(operator,part*operator)
part*operator	Var(Error) + Q(part*operator)

Tests of Hypotheses Using the Type III MS for part*operator as an Error Term					
Source	**DF**	**Type III SS**	**Mean Square**	**F Value**	**Pr > F**
operator	2	2.61666667	1.30833333	1.84	0.1730