Blocking in $2^k$ Factorial Designs

- For RCBD, each combination run in each block
  - $2^2 \rightarrow 4$ EUs per block
  - $2^3 \rightarrow 8$ EUs per block
  - Randomize run order within block
- Suppose you cannot run all combinations within block
- Must do some sort of incomplete block analysis
- If you do not, certain effects confounded
- Confounding: two effects are indistinguishable
- May “sacrifice” certain effects thought to be small
- $2^k$ design makes set-up simple

Confounding in $2^k$ with only 2 blocks

- Blocks assumed to allow $2^{k-1}$ combinations
- First consider $2^2$ factorial (2 combs per blk)
- Possible pairings
  1. (1) and b together → a and ab together
  2. (1) and a together → b and ab together
  3. (1) and ab together → a and b together
- Effect of A, $(ab+a-b-(1))/2$, is block difference
- Effect of B, $(ab-a+b-(1))/2$, is block difference
- Both have a main effect confounded with block
- Effect of AB, $(ab-a-b+1)/2$, is block difference
- Allows for main effect estimates (blks cancel out)

Choice of Confounding Factors

- Common to confound highest order interaction
- Can use +/- table to determine blks
- For two factor, recall the following table

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$AB$</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>+</td>
<td>(1)</td>
</tr>
<tr>
<td>+</td>
<td>-</td>
<td>-</td>
<td>a</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>b</td>
</tr>
<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>ab</td>
</tr>
<tr>
<td>+</td>
<td>-</td>
<td>-</td>
<td>c</td>
</tr>
<tr>
<td>+</td>
<td>-</td>
<td>+</td>
<td>ac</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>+</td>
<td>bc</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>abc</td>
</tr>
</tbody>
</table>

- Use confounding column to determine blks
- +’s in blk 1 and –’s in blk 2

- Consider three factor

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$AB$</th>
<th>$AC$</th>
<th>$BC$</th>
<th>$ABC$</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>(1)</td>
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<tr>
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<td>-</td>
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<td>+</td>
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<td>+</td>
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<td>+</td>
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<td>+</td>
<td>-</td>
<td>$b$</td>
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<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>$ab$</td>
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<tr>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$c$</td>
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<tr>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>$ac$</td>
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<tr>
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<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>$bc$</td>
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<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>$abc$</td>
</tr>
</tbody>
</table>

- Best assignment would be a,b,c,abc together
- Can estimate all but three factor interaction
**2^k** Factorial in Four Blocks

- Four blocks each containing 2^{k-2} EUs
- Useful in situations where k \geq 4
- Must select two effects to confound
- Will result in a third confounded factor
- Consider 6 factor factorial run in 4 blocks of 16 EUs
  - Block 1 uses ABC + and DEF +
  - Block 2 uses ABC + and DEF -
  - Block 3 uses ABC - and DEF +
  - Block 4 uses ABC - and DEF -
- Results in \((ABC)(DEF) = ABCDEFG\) confounded
  - ABCD and DEF \rightarrow ABCEF confounded
  - AB and ABEF \rightarrow EF confounded
- Can extend to 8 and 16 blks
- Table 7-8 summarizes these designs (pg 298)

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**Partial Confounding**

- Can replicate blocking design
- Confound different effects each replication
- Allows estimation of all effects
  - Confounded effects based on nonconfounded replicates
  - Can use Yates’ Algorithm for all nonconfounded effects
  - See Example 7-3 (pg 300)

```plaintext
\*[ Example 7-3 ]*
data cool;
  input block fact1 fact2 fact3 y;
cards;
  1 -1 -1 -1 -1 -3
  1 1 -1 -1 -1 -3
  ...
  ;
proc glm;
  class fact1 fact2 fact3 block;
  model y = block fact1 fact2 fact3;
```

---

**Example**

Consider a 2^3 factorial run in 4 blocks

Each replicate will result in 3 confounded effects

Consider 4 replicates for 32 total observations

Replicate 1: Confound BC and AC \rightarrow AB
Replicate 2: Confound BC and ABC \rightarrow A
Replicate 3: Confound AC and ABC \rightarrow B
Replicate 4: Confound AB and ABC \rightarrow C

Three replicates to estimate A, B, and C
Two replicates to estimate AB, AC, and BC
One replicate to estimate ABC
Since no effect is estimated from all replications (except error), we will compute the effect sums associated with each replicate and combine the appropriate information.

### Using SAS

```sas
options nocenter ps=50 ls=80;

data new;
  input repl blk a b c resp;
cards;
  1 1 0 0 0 75
  1 1 1 1 1 100
  1 2 1 1 0 89
  1 2 0 0 1 73
  ...
  4 3 0 0 1 50
  4 3 1 1 1 80
  4 4 1 0 1 37
  4 4 0 1 1 27
; proc glm;
class repl blk a b c;
model resp = repl blk(repl) a*b*c;
```

### SAS Output

**Dependent Variable: RESP**

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
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</thead>
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<td>1013.364583</td>
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<td>2420.0416667</td>
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<td>B</td>
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<td>0.1250000</td>
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</tbody>
</table>

**Source**

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<td>36481.13</td>
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<td>2420.0416667</td>
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