Blocking and Confounding in 2^k Factorial

Design of Experiments - Montgomery
Chapter 7

Blocking in 2^k Factorial Designs

- For RCBD, each combination run in each block
  - $2^2 \rightarrow 4$ EUs per block
  - $2^3 \rightarrow 8$ EUs per block
  - Randomize run order within block
- Suppose you cannot run all comb. within block
- Must do some sort of incomplete block analysis
- If you do not, certain effects confounded
- Confounding: two effects are indistinguishable
- May "sacrifice" certain effects thought to be small
- 2^k design makes set-up simple

Confounding in 2^k with only 2 blocks

- Blocks assumed to allow 2^{k-1} combinations
- First consider 2^2 factorial (2 combs per blk)
- Possible pairings
  1. (1) and b together $\rightarrow$ a and ab together
  2. (1) and a together $\rightarrow$ b and ab together
  3. (1) and ab together $\rightarrow$ a and b together

1 Effect of A, (ab+a-b-(1))/2, is block difference

2 Effect of B, (ab-a+b-(1))/2, is block difference

Both have a main effect confounded with block

3 Effect of AB, (ab-a-b+1)/2, is block difference

Allows for main effect estimates (blks cancel out)

Choice of Confounding Factors

- Common to confound highest order interaction
- Can use $+/-$ table to determine blks
- For two factor, recall the following table

A	B	AB	Symbol
-	-	+	(1)
+	-	-	a
-	+	-	b
+	+	+	ab

- Use confounding column to determine blks

  $+$'s in blk 1 and $-$'s in blk 2

- Consider three factor

A	B	C	AB	AC	BC	ABC	Symbol
-	-	-	+	+	+	-	(1)
+	-	-	-	-	+	+	a
-	+	-	-	+	-	+	b
+	+	-	+	-	-	-	ab
-	-	+	+	-	-	+	c
+	-	+	-	+	-	-	ac
-	+	+	-	-	+	-	bc
+	+	+	+	+	+	+	abc

  - Best assignment would be a,b,c,abc together
  - Can estimate all but three factor interaction

2^k Factorial in Four Blocks

- Four blocks each containing 2^{k-2} EUs
- Useful in situations where $k \geq 4$
- Must select two effects to confound
- Will result in a third confounded factor
- Consider 6 factor factorial run in 4 blocks of 16 EUs
  - Block 1 uses ABC + and DEF +
  - Block 2 uses ABC + and DEF -
  - Block 3 uses ABC - and DEF +
  - Block 4 uses ABC - and DEF -
- Results in (ABC)(DEF)=ABCDEF confounded
  - ABCD and DEF → ABCEF confounded
  - AB and ABEF → EF confounded
- Can extend to 8 and 16 blks
- Table 7-8 summarizes these designs (pg 298)

Partial Confounding

- Can replicate blocking design
- Confound different effects each replication
- Allows estimation of all effects
  - Confounded effects based on nonconfounded replicates
  - Can use Yates' Algorithm for all nonconfounded effects
  - See Example 7-3 (pg 300)

```
\* Example 7-3 *\
data cool;
input block fact1 fact2 fact3 y;
cards;
 1 -1 -1 -1 -3
 1  1  1 -1  2
...
;

proc glm;
class fact1 fact2 fact3 block;
model y= block fact1|fact2|fact3;
```



```
Class    Levels    Values
FACT1         2    1 -1
FACT2         2    1 -1
FACT3         2    1 -1
BLOCK         4    1 2 3 4

Dependent Variable: Y
Source                DF    Sum of Squares     F Value       Pr > F
Model                 10       74.25000000        9.90       0.0103
Error                  5        3.75000000
Corrected Total       15       78.00000000

Source                DF         Type I SS     F Value       Pr > F
BLOCK                  3        3.50000000        1.56       0.3101
FACT1                  1       36.00000000       48.00       0.0010
FACT2                  1       20.25000000       27.00       0.0035
FACT1*FACT2            1        0.50000000        0.67       0.4513
FACT3                  1       12.25000000       16.33       0.0099
FACT1*FACT3            1        0.25000000        0.33       0.5887
FACT2*FACT3            1        1.00000000        1.33       0.3004
FACT1*FACT2*FACT3      1        0.50000000        0.67       0.4513
```


Example

Consider a 2^3 factorial run in 4 blocks

Each replicate will result in 3 confounded effects

Consider 4 replicates for 32 total observations

Replicate 1: Confound BC and AC → AB

Replicate 2: Confound BC and ABC → A

Replicate 3: Confound AC and ABC → B

Replicate 4: Confound AB and ABC → C

Three replicates to estimate A, B, and C

Two replicates to estimate AB, AC, and BC

One replicate to estimate ABC

Data

Replicate 1 - AB, AC, BC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
75 (1)	89 ab	61 a	30 b
100 abc	73 c	45 bc	54 ac

Replicate 2 - A, BC, ABC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
60 (1)	47 a	1 b	26 ac
34 bc	81 abc	35 c	52 ab

Replicate 3 - B, AC, ABC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
58 (1)	48 a	18 b	68 ab
42 ac	52 c	82 abc	32 bc

Replicate 4 - C, AB, ABC Confounded

Blk 1	Blk 2	Blk 3	Blk 4
47 (1)	34 a	50 c	37 ac
57 ab	4 b	80 abc	27 bc

Since no effect is estimated from all replications (except error), we will compute the effect sums associated with each replicate and combine the appropriate information.

The following are the column 3 sums using Yates' Algorithm for each of the replicates.

Only the sums that are not confounded with the particular rep are presented

Effect	Rep 1	Rep 2	Rep 3	Rep 4
(1)	527	336	400	336
A	**81**	-	**80**	**80**
B	**1**	**0**	-	**0**
AB	-	**120**	**120**	-
C	**17**	**16**	**16**	-
AC	-	**0**	-	**0**
BC	-	-	**40**	**40**
ABC	**1**	-	-	-

$$SS_A = \frac{(81+80+80)^2}{3(8)}$$

$$SS_B = \frac{(1+0+0)^2}{3(8)}$$

$$SS_{AB} = \frac{(120+120)^2}{2(8)}$$

$$SS_C = \frac{(16+16+17)^2}{3(8)}$$

$$SS_{AC} = \frac{(0+0)^2}{2(8)}$$

$$SS_{BC} = \frac{(40+40)^2}{2(8)}$$

$$SS_{ABC} = \frac{1^2}{1(8)}$$

Using SAS

```
options nocenter ps=50 ls=80;

data new;
input repl blk a b c resp;
cards;
1 1 0 0 0 75
1 1 1 1 1 100
1 2 1 1 0 89
1 2 0 0 1 73
...
4 3 0 0 1 50
4 3 1 1 1 80
4 4 1 0 1 37
4 4 0 1 1 27
;

proc glm;
class repl blk a b c;
model resp = repl blk(repl) a|b|c;
```


SAS Output

```
Dependent Variable: RESP
                                  Sum of              Mean
Source                 DF        Squares            Square   F Value
Model                  22   17128.718750        778.578125  28028.81
Error                   9       0.250000          0.027778
Corrected Total        31   17128.968750

Source                 DF     Type I SS      Mean Square   F Value
REPL                    3   3040.0937500    1013.3645833  36481.13
BLK(REPL)              12   7568.3750000     630.6979167  22705.13
A                       1   2420.0416667    2420.0416667  87121.50
B                       1      0.0416667       0.0416667      1.50
A*B                     1   3600.0000000    3600.0000000  99999.99
C                       1    100.0416667     100.0416667   3601.50
A*C                     1      0.0000000       0.0000000      0.00
B*C                     1    400.0000000     400.0000000  14400.00
A*B*C                   1      0.1250000       0.1250000      4.50

Source                 DF   Type III SS      Mean Square   F Value
REPL                    3   3040.0937500    1013.3645833  36481.13
BLK(REPL)              12    653.9062500      54.4921875   1961.72
A                       1   2420.0416667    2420.0416667  87121.50
B                       1      0.0416667       0.0416667      1.50
A*B                     1   3600.0000000    3600.0000000  99999.99
C                       1    100.0416667     100.0416667   3601.50
A*C                     1      0.0000000       0.0000000      0.00
B*C                     1    400.0000000     400.0000000  14400.00
A*B*C                   1      0.1250000       0.1250000      4.50
```