Homework 1 – STAT 521.

Due on Sept. 2, 2015

1. Show that the set of polynomials of degree $\leq k$ forms a linear space over the field of real numbers.

2. Show that the null space of a matrix is closed under addition and scalar multiplication.

3. Let $v_i, i = 1, ..., n$ be an orthogonal basis for sample space Ω. Prove Parseval's Identity

$$\langle x, y \rangle = \sum_{i=1}^{n} \frac{\langle x, v_i \rangle \langle y, v_i \rangle}{\|v_i\|^2}, \forall x, y \in \Omega.$$

4. Let $V_1, ..., V_k$ be mutually orthogonal one-dimensional subspaces in Ω, none equal to $L(\mathbf{0})$. Prove that they are linearly independent.

5. Show that $B'BA = B'BC$ for matrices A, B and C with appropriate orders, is equivalent to $BA = BC$.

6. Show that if $rank(AB) = rank(A)$, then their range spaces are the same, $\mathcal{M}(AB) = \mathcal{M}(A)$.