Homework 2 – STAT 521.

Due on Sept. 18, 2015

1. If A, B are both nonegative definite matrcices, then $A+B$ is also nonnegative definite and $\mathcal{M}(A) \subset \mathcal{M}(A+B)$.

2. If $|A| \neq 0$, then it can be decomposed as $A = GH$ where G is a positive definite matrix and H is an orthogonal matrix.

3. Let $\lambda_1 \leq ... \leq \lambda_m$ be the solution of the determinant equation $|A - \lambda C| = 0$,with A symmetric and C positive definite. Show that

$$\inf_{X \in R^m} \frac{X^T AX}{X^T CX} = \lambda_1, \sup_{X \in R^m} \frac{X^T AX}{X^T CX} = \lambda_m$$

where X^T is the transpose of X.

4. Let V, W be two linear subspaces, show that $(V \cap W)^{\perp} = V^{\perp} + W^{\perp}$.

5. Let A be a $n \times m$ matrix with $rank(A) = r$. Then there exists a matrix B of order $s \times m$ such that $rank(B) = m - r$, and no overlap between the two row spaces i.e. $\mathcal{M}\left(A^T\right) \cap \mathcal{M}\left(B^T\right) = \{0\}$. Show that

(1). $A^T A + B^T B$ is of full rank m.

(2). $\left(A^T A + B^T B\right)^{-1}$ is a g-inverse of $A^T A$, i.e.

$$A^T A \left(A^T A + B^T B\right)^{-1} A^T A = A^T A.$$

6. For a real convex function ϕ, real numbers $a_1, a_2, ..., a_n$ in its domain, and positive weights w_i with $\sum_{i=1}^{n} w_i = 1$.

(1). Show that Jensen's inequality holds as follows:

$$\phi\left(\sum_{i=1}^{n} w_i a_i\right) \leq \sum_{i=1}^{n} w_i \phi(a_i).$$

(2). A discrete random variable X has probability mass function $p_i = P(X = x_i), i = 1, ..., k$. Show that it is true $\phi[E(X)] \leq E[\phi(X)]$.