# Homework 1 – Math 445 – Fall 08

Write up solutions for the exercises below.

1. Let $\mathcal{A}$ be the collection of subsets of $\mathbb{R}$ given by

$$\mathcal{A} := \{A \subset \mathbb{R} : \mathbb{R}\backslash A \text{ is finite}\} \cup \{\emptyset, \mathbb{R}\}.$$

   Prove:

   (a) If $A \in \mathcal{A}$ is such that $\mathbb{R}\backslash A \in \mathcal{A}$ then either $A = \emptyset$ or $A = \mathbb{R}$.

   (b) Arbitary unions and finite intersections of sets in $\mathcal{A}$ are in $\mathcal{A}$.

   (c) If $\mathcal{A}' \subset \mathcal{A}$ is such that $\mathbb{R} = \bigcup_{A \in \mathcal{A}'} A$ then there exist finitely many $A_1, \ldots, A_n \in \mathcal{A}'$ such that

$$\mathbb{R} = A_1 \cup \cdots \cup A_n.$$

2. Let $f : X \to Y$ be a function. Show that for any subset $B \subset Y$

$$f(f^{-1}(B)) \subset B. \tag{1}$$

   Find a function for which the inclusion in (1) is strict. Show that if $f$ is surjective then equality holds in (1).

3. Let $f : X \to Y$ be a function and $B_1, B_2 \subset Y$ subsets. Prove that

$$f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$$

   and

$$f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2).$$

4. Let $f : X \to Y$ be a function and $A_1, A_2 \subset X$ subsets. Prove that

$$f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$$

   and

$$f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2). \tag{2}$$

   Find a function for which the inclusion in (2) is strict.

**Due date:** Friday, September 5th, in class