

Homework 9 – Math 445 – Fall 2008

Write up solutions for the exercises below.

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}_\ell$ be given by $f(t) := |t|$ for all $t \in \mathbb{R}$. At which points is f continuous? Here, \mathbb{R} is given the standard topology and \mathbb{R}_ℓ the lower limit topology.
2. Let $f : \mathbb{R} \rightarrow \mathbb{R}_K$ be given by $f(t) := t$ for all $t \in \mathbb{R}$. At which points is f continuous? Here, \mathbb{R} is given the standard topology and \mathbb{R}_K the K -topology.
3. Let $f : \mathbb{R}_K \rightarrow \mathbb{R}$ be given by

$$f(t) := \begin{cases} 1 & t \in K \\ t & t \notin K. \end{cases}$$

Is f continuous at $t = 0$? Is f continuous at $t = \frac{1}{2}$? Recall that $K = \{1/n : n \in \mathbb{Z}_+\}$.

4. Let a topology τ on $X := [-1, 1]$ be given as follows: A subset $U \subset X$ is τ -open if and only if either $(-1, 1) \subset U$ or $0 \notin U$. Define a function $f : (0, 1) \rightarrow X$ by

$$f(t) := \begin{cases} 2t & 0 < t < \frac{1}{2} \\ 0 & t = \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} < t < 1. \end{cases}$$

At which points is f continuous? Here, $(0, 1)$ is endowed with the standard topology.

Due date: Monday, November 17th, in class