

## Homework 5 – Math 446 – Spring 09

Write up solutions for the exercises below.

1. Compute the fundamental group of the Moebius strip  $M^2$ . Proceed as follows:
  - (a) Show that  $\mathbb{R} \times [0, 1]$  is a covering space of  $M^2$ .
  - (b) Adapt the argument used to prove  $\pi_1(S^1) \cong \mathbb{Z}$  in order to find  $\pi_1(M^2)$ .
2. Show that the Moebius strip  $M^2$  is homeomorphic to  $\mathbb{RP}^2$  with a little disc removed.
3. Let  $f : [0, 1] \rightarrow S^2$  be a loop based at some points  $x_0 \in S^2$ . Show that if  $f$  omits a point of  $S^2$  then  $f \simeq_p e_{x_0}$ . (We will show in class that  $f \simeq_p e_{x_0}$  even when  $f([0, 1]) = S^2$ . In particular, this will show that  $S^2$  is simply connected.)
4. Use the fact (which we will prove in class) that  $S^2$  is simply connected to show that

$$\pi_1(\mathbb{RP}^2) \cong \mathbb{Z}_2.$$

Conclude that every covering space of  $\mathbb{RP}^2$  is compact. (We will show in class that in some sense the only covering spaces of  $\mathbb{RP}^2$  are  $S^2$  and  $\mathbb{RP}^2$  itself.)

5. Use the conclusions from exercise 4 to show that there exists no retraction of  $\mathbb{RP}^2$  to the subset  $A := \{[(\cos(\pi t), \sin(\pi t), 0)] : t \in [0, 1]\} \subset \mathbb{RP}^2$ .

This homework assignment will be collected and graded.

**Due date:** Wednesday, February 18th, 2009 (in class)