

## Homework 6 – Math 446 – Spring 09

Write up solutions for the exercises below.

1. Let  $X$  be a topological space and  $A, B \subset X$  subspaces. Show that if  $A$  and  $B$  are deformation retracts of  $X$  then  $A$  and  $B$  are homotopy equivalent.
2. Show that a space  $X$  is contractible if and only if every continuous map  $f : X \rightarrow Y$ , for arbitrary  $Y$ , is nulhomotopic.
3. Recall that the Klein bottle is the quotient space  $K^2 := [0, 1]^2 / \sim$  where  $(0, t) \sim (1, t)$  and  $(s, 0) \sim (1 - s, 1)$ .
  - (a) Show that  $\mathbb{R}^2$  is a covering space of  $K^2$ .
  - (b) Show that the fundamental group of  $K^2$  is not abelian.
4. Given groups  $G_1$  and  $G_2$  show that the multiplication in the free product  $G_1 * G_2$  is associative and thus  $G_1 * G_2$  is a group.
5. Solve exercises 1, 2, 4, 5, 6 in Section 58 of the textbook.

**Due date:** Wednesday, March 4th, 2009 (at 5pm)