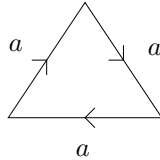


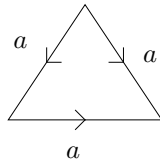
## Homework 7 – Math 446 – Spring 09

Write up solutions for the exercises below.

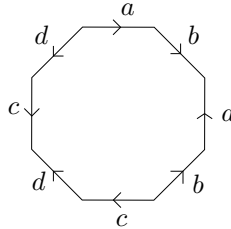
- Let  $X$  be the quotient space obtained from a triangle by identifying all its sides along the arrows indicated in the picture below. Compute the fundamental group of  $X$ .



- Let  $X$  be the quotient space obtained from a triangle by identifying all its sides along the arrows indicated in the picture below. Compute the fundamental group of  $X$ .



- Let  $X$  be the quotient space obtained from an 8-sided polygon with sides identified along the arrows indicated in the picture below. Show with a picture that  $X$  is homeomorphic to a torus with two holes. Compute the fundamental group of  $X$ .



- Let  $X$  be a path-connected topological space,  $x_1 \in X$ , and let  $\gamma : [0, 1] \rightarrow X$  be a continuous loop based at  $x_1$ . Let  $Y$  be the quotient space obtained from the union  $X \sqcup D^2$ , where  $D^2$  is the two-dimensional unit disc, and identifying  $e^{2\pi it}$  in  $D^2$  with  $\gamma(t)$ . Compute the fundamental group of  $Y$ .
- Let  $G$  and  $G'$  be two groups with presentations  $G = \langle S \mid \mathcal{R} \rangle$  and  $H = \langle S' \mid \mathcal{R}' \rangle$ . Define a group

$$W := \langle S \sqcup S' \mid \mathcal{R} \sqcup \mathcal{R}' \sqcup \mathcal{T} \rangle$$

where  $\mathcal{T} := \{ss's^{-1}s'^{-1} : s \in S, s' \in S'\}$ .

- Show that the map  $\psi : S \sqcup S' \rightarrow G \times G'$  given by  $\psi(s) := (s, e)$  for  $s \in S$  and  $\psi(s') := (e, s')$  for  $s' \in S'$  extends to a homomorphism  $\bar{\psi} : W \rightarrow G \times G'$ .
- Show that the map  $\eta : S \times S' \rightarrow W$  defined by  $\eta(s, s') := [ss']$  extends to a homomorphism  $\bar{\eta} : G \times G' \rightarrow W$ .
- Show that  $\bar{\psi} \circ \bar{\eta} = \text{id}_{G \times G'}$  and  $\bar{\eta} \circ \bar{\psi} = \text{id}_W$  and conclude that  $W$  and  $G \times G'$  are isomorphic.

**Due date:** Wednesday, March 11th, 2009 (at 5pm)