Worksheet # 2

MATH 294 ESP Workshop

Spring 2016

Problem 1. Let $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ and $\vec{u} = \begin{pmatrix} c \\ d \end{pmatrix}$ be vectors in $\mathbb{R}^2$. Recall that vector addition is defined according to the formula

$$\vec{v} + \vec{u} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}.$$

Vector multiplication is defined

$$\vec{v} \star \vec{u} = \begin{pmatrix} ac-bd \\ bc+ad \end{pmatrix}.$$

Use the definitions to prove the following about vector addition and multiplication in $\mathbb{R}^2$. Whenever there are no parenthesis we use the convention that multiplication is preformed before addition.

(1) For all $\vec{v}, \vec{u}, \vec{w} \in \mathbb{R}^2$,

$$(\vec{v} + \vec{u}) + \vec{w} = \vec{v} + (\vec{u} + \vec{w})$$

and

$$(\vec{v} \star \vec{u}) \star \vec{w} = \vec{v} \star (\vec{u} \star \vec{w}).$$

(2) For all $\vec{v}, \vec{u} \in \mathbb{R}^2$,

$$\vec{v} + \vec{u} = \vec{u} + \vec{v}$$

and

$$\vec{v} \star \vec{u} = \vec{u} \star \vec{v}.$$

(3) For all $\vec{v}, \vec{u}, \vec{w} \in \mathbb{R}^2$,

$$\vec{v} \star (\vec{u} + \vec{w}) = \vec{v} \star \vec{u} + \vec{v} \star \vec{w}$$

and

$$(\vec{v} + \vec{u}) \star \vec{w} = \vec{v} \star \vec{w} + \vec{u} \star \vec{w}.$$

(4) For all $\vec{v} \in \mathbb{R}^2$,

$$\vec{v} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \vec{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \vec{v}$$

and

$$\vec{v} \star \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \star v.$$

(5) For all $\vec{v} \in \mathbb{R}^2$ there is $\vec{u} \in \mathbb{R}^2$ so that

$$\vec{v} + \vec{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \vec{u} + \vec{v},$$

and for all $\vec{v} \in \mathbb{R}^2$ with $\vec{v} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ there is $\vec{u} \in \mathbb{R}^2$ so that

$$\vec{v} \star \vec{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \vec{u} \star \vec{v}.$$

(6) If $\vec{v} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\vec{u} = \begin{pmatrix} b \\ 0 \end{pmatrix}$ are both vectors whose second coordinate is zero, then their sum is $\begin{pmatrix} a+b \\ 0 \end{pmatrix}$ and their product is $\begin{pmatrix} ab \\ 0 \end{pmatrix}$.

(7)

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \star \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

(8) For any $\vec{v} \in \mathbb{R}^2$, $\vec{v}$ is perpendicular to $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \star \vec{v}$.

Problem 2. Simplify

$$\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \star \begin{pmatrix} b \\ 0 \end{pmatrix}$$

using properties and definitions from problem 1.

Problem 3. Let the norm of a vector $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$ be defined by $|\vec{v}| = \sqrt{a^2 + b^2}$. Prove for all $\vec{v}, \vec{u} \in \mathbb{R}^2$,

$$|\vec{v} \star \vec{u}| = |\vec{v}||\vec{u}|$$

where $\star$ is the multiplication defined in problem 1.
