

Worksheet # 4

MATH 294 ESP Workshop

Spring 2016

Problem 1. Discuss the following propositions. What do they mean? Which are true and which are false? Are there any you know how to prove or disprove?

- (1) Every real number has a unique decimal representation.
 - (2) Every natural number has a unique successor.
 - (3) There exists a largest prime number.
 - (4) There exists a smallest positive real number.
 - (5) Every function has a derivative.
 - (6) There is a function that has two different derivatives.
 - (7) For every pair of real numbers x, y , if $x < y$ then there is a real number z so that $x < z < y$.
 - (8) For every pair of integers x, y , if $x < y$ then there is an integer z so that $x < z < y$.
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Problem 2. Prove that the square of an even number is always even.

Problem 3. Let a, b , and c be integers with $a^2 + b^2 = c^2$. Prove at least one of a or b is even. Can both a and b be even?
