Math 220, Week 2 Thursday, Section 1.3-1.4

Spring 2020, XT

1.3 Direction Fields

1. Consider the differential equation

$$\frac{dp}{dt} = p(p-1)(2-p)$$

for the population p (in thousands) of a certain species at time t.

(a) Sketch the direction field.

(b) If the initial population is 4000, what is $\lim_{t\to+\infty} p(t)$?

(c) If $p(0) = 1.7$, what is $\lim_{t\to+\infty} p(t)$?

(d) If $p(0) = 0.8$, what is $\lim_{t\to+\infty} p(t)$?

(e) Can a population of 900 ever increase to 1100?

1.4 The Approximation Method of Euler

1. Use Euler's method to approximate the solution to the given initial problem at the points $x = 0.1, 0.2, 0.3, 0.4, 0.5$, $h = 0.1$.

$$\frac{dy}{dx} = \frac{x}{y}, \quad y(0) = -1$$

2. Given the Initial Value Problem

$$y' = x - y, \qquad y(0) = 0.$$

The actual solution is $y = e^{-x} + x - 1$. Find a value of h for Euler's method such that $y(1)$ is approximated to within ± 0.01.

Also find, to within ± 0.05, the value of x_0 such that $y(x_0) = 0.2$.