Math 220, Week 2 Tuesday, Section 1.2-1.3

Spring 2020, XT

1.2 Solutions and Initial Value Problems

1. Determine whether the given function is a solution to the given differential equation.
   $y = \sin x + x^2, \qquad \frac{d^2y}{dx^2} + y = x^2 + 2$

2. Determine whether the given relation is an implicit solution to the given differential equation.
   $e^{xy} + y = x - 1, \qquad \frac{dy}{dx} = \frac{e^{-xy} - y}{e^{-xy} + x}$

3. Verify that the function $\phi(x) = c_1 e^x + c_2 e^{-2x}$ is a solution to the linear equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

   for any choice of the constants c_1 and c_2. Determine c_1 and c_2 such that each of the following initial conditions is satisfied.

   (a) $y(0) = 2$, $y'(0) = 1$

   (b) $y(1) = 1$, $y'(1) = 0$

4. Determine whether the Theorem of Existence and Uniqueness of Solutions applies.
   $\frac{dy}{dt} - ty = \sin^2 t, \qquad y(\pi) = 5$

1.3 Direction Fields

1. Consider the differential equation

$$\frac{dp}{dt} = p(p-1)(2-p)$$

for the population p (in thousands) of a certain species at time t.

(a) Sketch the direction field.

(b) If the initial population is 4000, what is $\lim_{t\to+\infty} p(t)$?

(c) If $p(0) = 1.7$, what is $\lim_{t\to+\infty} p(t)$?

(d) If $p(0) = 0.8$, what is $\lim_{t\to+\infty} p(t)$?

(e) Can a population of 900 ever increase to 1100?