

Fourier Series

The **Fourier series** of f on the interval $[-L, L]$ is

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right).$$

where a_n and b_n are given by the formulas

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad n = 0, 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx \quad n = 1, 2, \dots$$

Fourier Cosine and Sine Series

The **Fourier cosine series** of $f(x)$ on $[0, L]$ is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right).$$

where

$$a_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad n = 0, 1, 2, \dots$$

The **Fourier sine series** of $f(x)$ on $[0, L]$ is

$$\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right).$$

where

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx \quad n = 1, 2, \dots$$

10.3 Fourier Series & 10.4 Fourier Cosine and Sine Series

1. Compute the Fourier series for function $f(x) = |x|$, $-\pi < x < \pi$.

2. Determine the π -periodic extension \tilde{f} , the odd 2π -periodic extension f_o and the odd 2π -periodic extension f_e for $f(x) = \pi - x$, $0 < x < \pi$.

3. Determine the Fourier sine series and cosine series for $f(x) = \pi - x$, $0 < x < \pi$.