2.2 Separable Equation

1. Solve the equation

(a) $\frac{dx}{dt} = \frac{t}{xe^{t+2x}} = \frac{t}{xe^t e^{2x}} = \frac{t}{e^t} \cdot \frac{1}{xe^{2x}}$

Separate: $xe^{2x}dx = te^{-t}dt$

~~I.B.P.~~ $\int xe^{2x}dx = \int te^{-t}dt$ Integrate

Simplify: $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} = -te^{-t} - e^{-t} + C$

Integration by parts: $udv = uv - vdu$

Left: $u = x$ $du = dx$
$dv = e^{2x}dx$ $v = \frac{1}{2}e^{2x}$

$\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx \Rightarrow \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$

Right: $u = t$ $du = dt$
$dv = e^{-t}dt$ $v = -e^{-t}$

$-te^{-t} - (-e^{-t})dt \Rightarrow -te^{-t} - e^{-t}$

2. Solve the initial value problem

(a) $\frac{1}{2}\frac{dy}{dx} = \sqrt{y+1}\cos x, \qquad y(\pi) = 0$

Separate: $\frac{1}{2}\frac{1}{\sqrt{y+1}}dy = \cos x\, dx$

Integrate: $\int \frac{1}{2}\frac{1}{\sqrt{y+1}}dy = \int \cos dx$

$(y+1)^{\frac{1}{2}} = \sin x + C$

Use $y(\pi) = 0 \Rightarrow x = \pi,\ y = 0$

$(0+1)^{\frac{1}{2}} = \sin\pi + C \Rightarrow C = 1$

$(y+1)^{\frac{1}{2}} = \sin x + 1$

$y = (\sin x + 1)^2 - 1$

(b) $x^2dx + 2ydy = 0, \qquad y(0) = 2$

Separate: $x^2dx = -2ydy$

Integrate: $\int x^2dx = \int -2ydy$

$\frac{1}{3}x^3 = -\frac{y^2}{2} + C$

$x = 0,\ y = 2$

$0 = -\frac{4}{2} + C \Rightarrow C = 4$

$-y^2 + 4 = \frac{1}{3}x^3$

$y^2 = 4 - \frac{1}{3}x^3 \Rightarrow y = \sqrt{4 - \frac{1}{3}x^3}$

$\cancel{y = -\sqrt{4 - \frac{1}{3}x^3}}$ Exclude b/c: $y(0) = 2$ not $y(0) = -2$