

2.4 Exact Equations

1. Determine whether the equation is exact. If it is, then solve it.

$$\underline{M} \quad \underline{N}$$

$$(2xy + 3)dx + (x^2 - 1)dy = 0$$

check: $\frac{\partial M}{\partial y} = 2x, \frac{\partial N}{\partial x} = 2x \quad \checkmark \text{ Yes, it is exact.}$

1. use $\frac{\partial F}{\partial x} = M = 2xy + 3$ solve for $F(x,y) = \int (2xy + 3)dx = x^2y + 3x + C(y)$

2. use $\frac{\partial F}{\partial y} = N = x^2 - 1$ solve for $F(x,y) = \int (x^2 - 1)dy = x^2y - y + D(x)$

3. Compare: $F(x,y) = x^2y + 3x + C(y) = x^2y - y + D(x)$. $C(y) = -y, D(x) = 3x$

4. Set $F(x,y) = C \Rightarrow$ solution: $x^2y + 3x - y = C$

2. Solve the initial value problem

$$\underline{M} \quad \underline{N}$$

$$(e^t x + 1)dt + (e^t - 1)dx = 0, \quad x(1) = 1 \quad (t, x)$$

check: $\frac{\partial M}{\partial y} = e^t, \frac{\partial N}{\partial x} = e^t \quad \checkmark \text{ Yes, it is exact}$

1. $\frac{\partial F}{\partial t} = M = e^t x + 1 \Rightarrow F(t, x) = \int (e^t x + 1)dt = e^t x + t + C(x)$

2. $\frac{\partial F}{\partial x} = N = e^t - 1$ use $\frac{\partial F}{\partial x} = e^t + 0 + C'(x) = e^t - 1$

$$\Rightarrow C'(x) = -1 \quad C(x) = \int (-1)dx = -x$$

3. Set $F(t, x) = e^t x + t - x = C$

4. Use $x(1) = 1, x=1, t=1$, solve for C

$$F(1, 1) = e + 1 - 1 = e = C \Rightarrow C = e$$

Sol $e^t x + t - x = e \quad \text{or} \quad x(t) = \frac{e^{-t}}{e^t - 1}$