

* You must write down the guess of $y = e^{rt}$ *

4.2 Homogeneous Linear Equations

y_h - represent the homogeneous solutions

- Find a general solution to the differential equations

$$(a) 2y'' + 7y' - 4y = 0$$

Guess $y = e^{rt}$

Auxiliary Eqn: $2r^2 + 7r - 4 = 0$

$$(2r-1)(r+4) = 0 \Rightarrow r_1 = \frac{1}{2}, r_2 = -4$$

Homogeneous soln $y_1 = e^{\frac{1}{2}t}$ $y_2 = e^{-4t}$

$$\text{General sol: } y_h(t) = C_1 y_1 + C_2 y_2 = C_1 e^{\frac{1}{2}t} + C_2 e^{-4t}$$

- Solve the initial value problem

$$(a) y'' - 4y' + 3y = 0; \quad y(0) = 1, \quad y'(0) = 1/3$$

$$(b) 4y'' - 4y' + y = 0$$

Guess $y = e^{rt}$

$$4r^2 - 4r + 1 = 0$$

$$(2r-1)(2r-1) = 0 \quad r_1 = r_2 = \frac{1}{2}$$

Double roots (repeated)

$$y_1 = e^{\frac{1}{2}t} \quad y_2 = te^{\frac{1}{2}t}$$

$$\text{Sol: } y_h(t) = C_1 e^{\frac{1}{2}t} + C_2 t e^{\frac{1}{2}t}$$

$$r^2 - 4r + 3 = 0 \quad (r-3)(r-1) = 0 \quad r_1 = 3, \quad r_2 = 1 \Rightarrow y_1(t) = e^{3t} \quad y_2(t) = e^t$$

$$\text{General sol: } y_h(t) = C_1 e^{3t} + C_2 e^t \quad y_h' = 3C_1 e^{3t} + C_2 e^t$$

$$\begin{cases} \text{Initial conditions} \\ y(0) = C_1 + C_2 = 1 \\ y'(0) = 3C_1 + C_2 = \frac{1}{3} \end{cases} \quad \begin{cases} \text{Solve: } C_1 = -\frac{1}{3} \\ C_2 = \frac{4}{3} \end{cases}$$

$$\text{Sol: } y(t) = -\frac{1}{3}e^{3t} + \frac{4}{3}e^t$$

$$(b) y'' - 4y' + 4y = 0 \quad y(1) = 1, \quad y'(1) = 1 \quad (\text{GR})$$

Double roots: $r_1 = r_2 = 2$

$$y_1(t) = e^{2t} \quad y_2(t) = te^{2t}$$

$$y_h(t) = C_1 e^{2t} + C_2 t e^{2t}$$

$$\text{Initial values: } y'(t) = 2C_1 e^{2t} + C_2 e^{2t} + 2C_2 t e^{2t}$$

$$\begin{cases} y(1) = C_1 e^2 + C_2 e^2 = 1 \\ y'(1) = 2C_1 e^2 + 3C_2 e^2 = 1 \end{cases}$$

$$\begin{cases} C_1 = 2e^{-2} \\ C_2 = -e^{-2} \end{cases}$$

$$\begin{cases} y(t) = 2e^{-2}e^{2t} - e^{-2}te^{2t} \\ y(t) = 2e^{2t-2} - te^{2t-2} \end{cases}$$

- Determine whether the functions y_1 and y_2 are linearly dependent on $(0, 1)$

$$(a) y_1(t) = \cos t \sin t, \quad y_2(t) = \sin 2t$$

$$y_1(t) = \cos t \sin t = \frac{1}{2} \sin(2t) = \frac{1}{2} y_2(t)$$

Linearly dependent

USE definition 1 in Textbook

$$(b) y_1(t) = 0, \quad y_2(t) = e^t$$

$$y_1(t) = 0 \cdot y_2(t) \quad \text{Linearly dependent:}$$

$$\overset{\uparrow}{C=0}$$

Function $y(t) = 0$ is linearly dependent with others.