

7.2 Definition of Laplace Transform

Use definition to determine the Laplace transform

$$1. f(t) = \begin{cases} 1-t, & 0 < t < 1 \\ 0, & 1 < t \end{cases}$$

7.3 Properties of Laplace Transform

1. Use the table and properties to determine their Laplace transforms

(a) $\mathcal{L}\{t^2 - 3t - 2e^{-t} \sin 3t\}$

(b) $\mathcal{L}\{e^{-2t} \cos \sqrt{3}t - t^2 e^{-2t}\}$

2. Use the table and properties to determine their Laplace transforms

(a) $te^{2t} \cos 5t$

(b) $\sin^2 t$

(c) $\sin(t - \pi)$

7.4 Inverse Laplace Transform

1. Use the Laplace transform table to determine their inverse Laplace transforms

(a) $\frac{1}{s^2 + 4s + 8} = F(s)$

(a) $F(s) = \frac{1}{(s+2)^2 + 4} = \frac{1}{(s+2)^2 + 2^2}$

$$= \frac{2}{(s - (-2))^2 + 2^2} \cdot \frac{1}{2}$$

$$\mathcal{L}^{-1}\{F(s)\}(t) = \frac{1}{2} \cdot e^{-2t} \sin 2t$$

F7, $a = -2, b = 2$

(b) $\frac{3}{(2s+5)^3} = F(s)$

(b) $F(s) = \frac{3}{2^3 (s + \frac{5}{2})^3}$

$$= \frac{3 \cdot 2!}{2^3 [s - (-\frac{5}{2})]^{2+1}} \cdot \frac{1}{2!}$$

$$= \frac{3}{2^3} \cdot \frac{1}{2} \cdot \frac{2!}{[s - (-\frac{5}{2})]^{2+1}}$$

$$\mathcal{L}^{-1}\{F(s)\}(t) = \frac{3}{8} \cdot \frac{1}{2} \cdot t^2 e^{-\frac{5}{2}t}$$

F6, $n=2, a = -\frac{5}{2} = \frac{3}{16} t^2 e^{-\frac{5}{2}t}$

7.2 Use definition

$$\mathcal{L}\{f(t)\}(s) = \int_0^{\infty} e^{-st} f(t) dt = \int_0^1 e^{-st} (1-t) dt + \int_1^{\infty} e^{-st} \cdot 0 dt$$

$$= \int_0^1 e^{-st} (1-t) dt \stackrel{\text{IBP}}{=} (1-t) \frac{1}{-s} e^{-st} \Big|_{t=0}^{t=1} - \int_0^1 \frac{1}{-s} e^{-st} (-dt)$$

$$\text{IBP: } \left. \begin{array}{l} u = (1-t) \quad du = -dt \\ dv = e^{-st} \quad v = \frac{1}{-s} e^{-st} \end{array} \right\} = 0 - \frac{1}{s} e^0 - \frac{1}{s} \frac{1}{-s} e^{-st} \Big|_{t=0}^{t=1} = \frac{1}{s} + \frac{1}{s^2} (e^{-s} - e^0)$$

$$= \frac{1}{s} + \frac{e^{-s}}{s^2} - \frac{1}{s^2} \quad \boxed{s \neq 0}$$

If $s=0$, $\mathcal{L}\{f(t)\}(0) = \int_0^{\infty} f(t) dt = \int_0^1 (1-t) dt + 0 = t - \frac{t^2}{2} \Big|_0^1 = \frac{1}{2}$

$$\mathcal{L}\{f(t)\}(s) = \begin{cases} \frac{1}{s} + \frac{e^{-s}}{s^2} - \frac{1}{s^2} & s \neq 0 \\ \frac{1}{2} & s = 0 \end{cases}$$

7.3.1(a). $\mathcal{L}\{t^2 - 3t - 2e^{-t} \sin 3t\}(s) = \frac{2!}{s^{2+1}} - 3 \frac{1!}{s^{1+1}} - 2 \frac{3}{(s-(-1))^2 + 3^2} = \frac{2}{s^3} - \frac{3}{s^2} - \frac{6}{(s+1)^2 + 9}, s > 0$

FB, $n=2$, F3, $n=1$, F7, $a=-1$, $b=3$ $s > 0$ $s > 0$ $s > -1$

(b) $\mathcal{L}\{e^{-2t} \cos 3t - t^2 e^{-2t}\}(s) = \frac{s - (-2)}{(s - (-2))^2 + 3^2} - \frac{2!}{(s - (-2))^{2+1}} = \frac{s+2}{(s+2)^2 + 3} - \frac{2}{(s+2)^3}, s > -2$

F8, $a=-2$, $b=3$ F6, $a=-2$, $n=2$ $s > -2$ $s > -2$

2(a) $\mathcal{L}\{t e^{2t} \cos 3t\}(s) = \mathcal{L}\{t \cdot e^{2t} \cos 3t\}(s) \stackrel{\text{F13}}{=} (-1)^1 \frac{d}{ds} \mathcal{L}\{e^{2t} \cos 3t\}(s) \stackrel{\text{F8}}{=} -\frac{d}{ds} \left(\frac{s-2}{(s-2)^2 + 5^2} \right) \quad s > 2$

$$= -\frac{(s-2)^2 + 25 - (s-2)2(s-2)}{[(s-2)^2 + 25]^2} = -\frac{-(s-2)^2 + 25}{[(s-2)^2 + 25]^2} \quad s > 2$$

(b) $\sin^2 t = \frac{1 - \cos 2t}{2} = \frac{1}{2} - \frac{1}{2} \cos 2t$

$$\mathcal{L}\{\sin^2 t\}(s) = \mathcal{L}\left\{\frac{1}{2}\right\}(s) - \mathcal{L}\left\{\frac{1}{2} \cos 2t\right\}(s) = \frac{1}{2} \frac{1}{s} - \frac{1}{2} \frac{s}{s^2 + 4} = \frac{1}{2s} - \frac{s}{2(s^2 + 4)}, s > 0$$

F1 F5 $s > 0$ $s > 0$

(c) $\sin(t - \pi) = -\sin t$

$$\mathcal{L}\{\sin(t - \pi)\}(s) = \mathcal{L}\{-\sin t\}(s) = -\frac{1}{s^2 + 1}, s > 0$$

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