

## 7.4 Inverse Laplace Transform

1. Use method of partial fractions to determine inverse Laplace Transform  $\mathcal{L}^{-1}\{F\}$

Discussed  
in lecture (a)  $F(s) = \frac{s+11}{(s-1)(s+3)} = \frac{A}{s-1} + \frac{B}{s+3}$

Multiply by  $(s-1)(s+3)$ :  $s+11 = A(s+3) + B(s-1)$

$$s=1 \quad 12 = A(4) \quad A=3$$

$$s=-3 \quad 8 = B(-4) \quad B=-2$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{3}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{-2}{s+3}\right\} = 3e^t - 2e^{-3t}$$

$$F_2, a=1 \quad F_2, a=-3$$

(b)  $F(s) = \frac{5s^2 + 34s + 53}{(s+3)^2(s+1)} = \frac{A}{s+3} + \frac{B}{(s+3)^2} + \frac{C}{s+1}$

$$5s^2 + 34s + 53 = A(s+3)(s+1) + B(s+1) + C(s+3)^2$$

$$s=-3 \quad 45-102+53 = -2B \quad -4 = -2B \quad B=2$$

$$s=-1 \quad 5-34+53 = C(4) \quad 24 = 4C \quad C=6$$

$$s=0 \quad 53 = A(3) + B(1) + C(9) = 3A + 2 + 54 \\ -3 = 3A \quad A = -1$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{-1}{s+3}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s+3)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{6}{s+1}\right\}$$

$F_2, a=-3 \quad F_6, n=1, a=-1 \quad F_2, a=-1$   
 $2 \cdot \frac{1!}{(s-2-3)^{1+1}}$

$$= -e^{-3t} + 2t e^{-3t} + 6e^{-t}$$

$$(c) F(s) = \frac{7s^2 - 41s + 84}{(s-1)(s^2 - 4s + 13)} = \frac{A}{s-1} + \frac{Bs + C}{s^2 - 4s + 13}$$

irreducible  $\rightarrow (s-2)^2 + 3^2$

$$7s^2 - 41s + 84 = A(s^2 - 4s + 13) + (Bs + C)(s-1)$$

$$s=1 \quad 7-41+84 = A(10) \quad 50 = 10A \quad A=5$$

$$s=0 \quad 84 = A(13) + C(-1)$$

$$84 = 65 - C \quad C = -19$$

$$s=2 \quad 28 - 82 + 84 = A(9) + (2B + C)$$

$$30 = 45 + 2B - 19 \quad 4 = 2B \quad B = 2$$

$$F(s) = \frac{5}{s-1} + \frac{2s-19}{s^2 - 4s + 13} = \frac{5}{s-1} + \frac{2(s-2) + 4 - 19}{(s-2)^2 + 3^2}$$

$$= \frac{5}{s-1} + \frac{2(s-2)}{(s-2)^2 + 3^2} + \frac{(-5)3}{(s-2)^2 + 3^2}$$

$$F_2, a=1 \quad F_8, a=2, b=3 \quad F_7, a=2, b=3$$

$$\mathcal{L}^{-1}\{F(s)\} = 5e^t + 2e^{2t} \cos 3t - 5e^{2t} \sin 3t.$$