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Counterexample to boundary regularity of a strongly pseudoconvex CR submanifold: An addendum to the paper of Harvey-Lawson

By HING SUN LUK and STEPHEN S.-T. YAU*

The purpose of this paper is to give a counterexample of Theorem 10.4 in [Ha-La]. In the Harvey-Lawson paper, a global result is claimed, but only a local result is proven. This theorem has had a big impact on CR geometry for almost a quarter of a century because one can use the theory of isolated singularities to study the theory of CR manifolds and vice versa.

Example. Consider the following holomorphic map:

$$\begin{aligned} F : \mathbf{C}^2 &\longrightarrow \mathbf{C}^3 \\ (u, v) &\longrightarrow (x, y, z) = (u(u-1), v, u^2(u-1)). \end{aligned}$$

Clearly for any c , F restricted on the line $\{v = c\}$ is an embedding outside the two points $(0, c)$ and $(1, c)$. F sends $(0, t)$ and $(1, t)$ to $(0, t, 0)$ for all t . Now take S , which is the boundary of a ball $B = \{(u, v) \in \mathbf{C}^2 : \|(u, v)\| \leq 2\}$. It is easy to see that the mapping F restricted on S is still an embedding. The image of S under F is a strongly pseudoconvex CR manifold in \mathbf{C}^3 . The variety that $F(S)$ bounds is $F(B)$. Observe that $F(B)$ has curve singularities along the line $(0, t, 0)$. We remark that $F(\mathbf{C}^2)$ is a hypersurface $\{(x, y, z) \in \mathbf{C}^3 : z^2 - zx - x^3 = 0\}$ in \mathbf{C}^3 .

Theorem 10.4 of [Ha-La] was so powerful that it has been used by many researchers. Fortunately, we can replace it by the following theorem, the proof of which will appear elsewhere [Lu-Ya].

THEOREM. *Let X be a strongly pseudoconvex CR manifold of dimension $2n - 1$, $n \geq 2$. If X is contained in the boundary of a bounded strictly pseudoconvex domain D in \mathbf{C}^N , then there exists a complex analytic subvariety V of dimension n in $D - X$ such that the boundary of V is X . Moreover, V has boundary regularity at every point of X , and V has only isolated singularities in $V|X$.*

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