J. DIFFERENTIAL GEOMETRY
63 (2003) 155-170

HOLOMORPHIC DE RHAM COHOMOLOGY OF STRONGLY PSEUDOCONVEX CR MANIFOLDS WITH S^1-ACTIONS

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This paper is dedicated to Professor Yum Tong Siu on the occasion of his 60th birthday.

Abstract

In this paper, we study the holomorphic de Rham cohomology of a compact strongly pseudoconvex CR manifold X in $\mathbb{C}^N$ with a transversal holomorphic S^1-action. The holomorphic de Rham cohomology is derived from the Kohn-Rossi cohomology and is particularly interesting when X is of real dimension three and the Kohn-Rossi cohomology is infinite dimensional. In Theorem A, we relate the holomorphic de Rham cohomology $H_h^k(X)$ to the punctured local holomorphic de Rham cohomology at the singularity in the variety V which X bounds. In case X is of real codimension three in $\mathbb{C}^{n+1}$, we prove that $H_h^{n-1}(X)$ and $H_h^n(X)$ have the same dimension while all other $H_h^k(X)$, $k > 0$, vanish (Theorem B). If X is three-dimensional and V has at most rational singularities, we prove that $H_h^1(X)$ and $H_h^2(X)$ vanish (Theorem C). In case X is three-dimensional and $N = 3$, we obtain in Theorem D a complete characterization of the vanishing of the holomorphic de Rham cohomology of X.

1. Introduction

Let M be a complex manifold. The k-th holomorphic de Rham cohomology $H_h^k(M)$ of M is defined to be the d-closed holomorphic k-forms quotient by the d-exact holomorphic k-forms. It is well-known that if M is a Stein manifold, then the holomorphic de Rham cohomology coincides with the ordinary de Rham cohomology. For any CR manifold

Research partially supported by NSF.
Research partially supported by RGC Hong Kong.
Received 01/11/2002.

X, one can define the holomorphic de Rham cohomology group $H_h^k(X)$. One may regard the holomorphic de Rham cohomology as derived from the Kohn-Rossi cohomology, in the sense, following Tanaka [9], that the Kohn-Rossi cohomology groups are the $E_1^{p,q}(X)$ terms and the holomorphic de Rham cohomology groups are the $E_2^{k,0}(X)$ terms of a natural spectral sequence $E_r^{p,q}(X)$ on X.

Let X be a compact connected strongly pseudoconvex CR manifold which admits a transversal holomorphic S^1-action φ_t. Suppose that X is of dimension $2n-1$ ($n \geqslant 2$) and is CR embeddable in complex Euclidean space. Then it is known [5] that X bounds a compact complex analytic variety V and φ_t extends to a weakly holomorphic $\triangle^*$-action Φ_t on V with a single fixed point x where $\triangle^* = \{z \in \mathbb{C} : 0 < |z| \leqslant 1\}$. Further, V has at most a singularity at x. Thus a natural problem is to study how the holomorphic de Rham cohomology of X is related to invariants at the point x.

In [3], we introduce the punctured local holomorphic de Rham cohomology $H_h^k(V,x)$ at any point x in a complex analytic space V with only isolated singularities. The punctured local holomorphic de Rham cohomology vanishes at regular points. It turns out that the punctured local holomorphic de Rham cohomology is an important local invariant which can be used to tell when an isolated singularity is quasi-homogeneous. Our first main result in this paper is the following theorem.

Theorem A. *Let X be a compact connected $(2n-1)$-dimensional $(n \geqslant 2)$ strongly pseudoconvex* CR *manifold with a transversal holomorphic S^1-action. Suppose that X bounds a complex analytic variety V in $\mathbb{C}^N$ with x as the only singular point of V. Then the holomorphic de Rham cohomology of X is naturally isomorphic to the punctured local holomorphic de Rham cohomology of V at x. That is, $H_h^k(X) \cong H_h^k(V,x)$ for all $k \geqslant 0$.*

Theorem A is proved by a homotopy argument, which works also when the fixed point x of the extended $\triangle^*$-action Φ_t is a regular point of V, showing that $H_h^k(X) = 0$ for all $k \geqslant 1$ in such cases. By Theorem A and results in [3] on punctured local holomorphic de Rham cohomology, we get

Theorem B. *Let X be a compact connected $(2n-1)$-dimensional $(n \geqslant 2)$ strongly pseudoconvex* CR *manifold with a transversal holomorphic S^1-action. Suppose that X bounds a compact complex analytic hypersurface V in $\mathbb{C}^{n+1}$. Then:*

(1) $\dim H_h^k(X) = 0 \qquad 1 \leqslant k \leqslant n-2.$

(2) $\dim H_h^{n-1}(X) = \dim H_h^n(X).$

The holomorphic de Rham cohomology is particularly interesting when X is three dimensional, in which case the Kohn-Rossi cohomology is infinite dimensional. The simplest singularities that a three dimensional X may bound in $\mathbb{C}^3$ are the rational singularities of types A_n, D_n, E_6, E_7, E_8 and we prove the following theorem.

Theorem C. *Let X be a compact connected three dimensional strongly pseudoconvex* CR *manifold with transversal holomorphic S^1-action. Suppose that X bounds a compact complex analytic variety V in $\mathbb{C}^N$ with at most rational singularities. Then $H_h^1(X) = 0$ and $H_h^2(X) = 0$.*

We would like to remark that in the statement of Theorem C, X is not required to be in $\mathbb{C}^3$ and the singularity in V may not be of types A_n, D_n, E_6, E_7 and E_8. If X is in $\mathbb{C}^3$, then the following theorem gives a complete characterization of when the holomorphic de Rham cohomology groups of X vanish.

Theorem D. *Let X in $\mathbb{C}^3$ be a compact connected 3-dimensional strongly pseudoconvex* CR *manifold with transversal holomorphic S^1-action. Then X bounds a compact complex analytic variety V in $\mathbb{C}^3$ with at most one singularity. Let M be a resolution of V with A as exceptional set. Then $H_h^1(X) = 0$ and $H_h^2(X) = 0$ if and only if $H^1(A, \mathbb{C}) = 0$.*

Our assumption of the existence of a transversal holomorphic S^1-action on the CR manifold is a natural assumption arising from the work of Lawson and Yau [5]. Such CR manifolds include the intersection of any quasihomogeneous hypersurface in $\mathbb{C}^{n+1}$ with a sphere centered at the origin.

In Section 2, we recall the intrinsic definition of the holomorphic de Rham cohomology of CR manifolds and clarify the extrinsic formulation in the boundary case (Theorem 2.7). Theorems A, B, C and D are proved in Section 3. We prove Theorem C using a proposition of Campana and Flenner [2].

Part of this work was done while the second author was a Ze Jiang Visiting Professor at East China Normal University.

2. Holomorphic de Rham cohomology of CR manifolds

Kohn-Rossi cohomology was first introduced by Kohn-Rossi [4]. Following Tanaka [9], we reformulate the definition in a way independent of the interior manifold and derive the holomorphic de Rham cohomology from the Kohn-Rossi cohomology.

Definition 2.1. Let X be a connected orientable manifold of real dimension $2n-1$, $n \geqslant 2$. A CR structure on X is a rank $n-1$ subbundle S of the complexified tangent bundle $\mathbb{C}T(X)$ such that:

(1) $S \cap \overline{S} = \{0\}$.

(2) If L, L' are local sections of S, then so is $[L, L']$.

A manifold with a CR structure is called a CR manifold. Then there is a unique subbundle $\mathcal{H}$ of $T(X)$ and a unique homomorphism $J : \mathcal{H} \longrightarrow \mathcal{H}$ such that $\mathbb{C}\mathcal{H} = S \oplus \overline{S}$, $J^2 = -1$ and $S = \{v - iJv : v \in \mathcal{H}\}$.

For any C^∞ function u, there is a section $\overline{\partial}_b u \in \Gamma(\overline{S}^*)$ defined by $(\overline{\partial}_b u)(\overline{L}) = \overline{L}u$ for any $L \in \Gamma(S)$. This can be generalized as follows:

Definition 2.2. A complex vector bundle E over X is said to be holomorphic if there is a differential operator $\overline{\partial}_E : \Gamma(E) \longrightarrow \Gamma(E \otimes \overline{S}^*)$ such that if $\overline{L}u$ denotes $(\overline{\partial}_E u)(\overline{L})$ for $u \in \Gamma(E)$ and $L \in \Gamma(S)$, then for any $L_1, L_2 \in \Gamma(S)$ and any C^∞ function f on X:

(1) $\overline{L}(fu) = (\overline{L}f)u + f(\overline{L}u)$.

(2) $[\overline{L}_1, \overline{L}_2]u = \overline{L}_1\overline{L}_2 u - \overline{L}_2\overline{L}_1 u$.

A solution u of the equation $\overline{\partial}_E u = 0$ is called a holomorphic section.

The vector bundle $\hat{T}(X) = \mathbb{C}T(X)/\overline{S}$ is holomorphic with respect to the following $\overline{\partial} = \overline{\partial}_{\hat{T}(X)}$. Let ω be the projection from $\mathbb{C}T(X)$ to $\hat{T}(X)$. Take any $u \in \Gamma(\hat{T}(X))$ and express it as $u = \omega(Z)$, $Z \in \Gamma(\mathbb{C}T(X))$. For any $L \in \Gamma(S)$, define $(\overline{\partial}u)(\overline{L}) = \omega([\overline{L}, Z])$. The section $(\overline{\partial}u)(\overline{L})$ of $\hat{T}(X)$ does not depend on the choice of Z and $\overline{\partial}u$ gives a section of $\hat{T}(X) \otimes \overline{S}^*$. Further the operator $\overline{\partial}$ satisfies the conditions in Definition 2.2. The resulting holomorphic vector bundle $\hat{T}(X)$ is called the holomorphic tangent bundle of X.

Lemma 2.3. *If X is a real hypersurface in a complex manifold M, then the holomorphic tangent bundle $\hat{T}(X)$ is naturally isomorphic to the restriction to X of the bundle $T^{1,0}(M)$ of all $(1,0)$ tangent vectors to M.*

Proof. The CR structure of X is given by $S = \mathbb{C}T(X) \cap T^{1,0}(M)$. The inclusion $\mathbb{C}T(X) \longrightarrow \mathbb{C}T(M)$ induces a vector bundle isomorphism $\mathbb{C}T(X)/\overline{S} \longrightarrow \mathbb{C}T(M)/T^{0,1}(M)$, which we write as $\Phi : \hat{T}(X) \longrightarrow T^{1,0}(M)|_X$. $T^{1,0}(M)|_X$ is a holomorphic vector bundle with respect to the following natural $\overline{\partial}$ operator. For $L \in \Gamma(S)$ and $Y \in \Gamma(T^{1,0}(M)|_X)$, $(\overline{\partial}Y)(\overline{L}) = \overline{L}(Y)$ is defined to be the $(1,0)$ component of $[\overline{L}, Y]$. If $Y = \sum y^i \dfrac{\partial}{\partial z^j}$ in local coordinates of M, then $\overline{L}(Y) = \sum(\overline{L}y^j)\dfrac{\partial}{\partial z^j}$. It follows that Φ is an isomorphism of holomorphic vector bundles in the obvious sense that $\overline{L}\Phi u = \Phi\overline{L}u$ for any $u \in \Gamma(\widehat{T}(X))$. q.e.d.

For a holomorphic vector bundle E over X, set

$$C^q(X,E) = E \otimes \bigwedge\nolimits^q \overline{S}^*, \qquad \mathcal{C}^q(X,E) = \Gamma(C^q(X,E)).$$

$\overline{\partial}_E$ induces a differential operator $\overline{\partial}_E^q : \mathcal{C}^q(X,E) \longrightarrow \mathcal{C}^{q+1}(X,E)$ as follows. For any $\psi \in \mathcal{C}^q(X,E)$, $(\overline{\partial}_E^q\psi)(\overline{L}_1, \ldots, \overline{L}_{q+1}) = \sum_i(-1)^{i+1} \overline{L}_i(\psi(\overline{L}_1, \ldots, \hat{\overline{L}}_i, \ldots, \overline{L}_{q+1})) + \sum_{i<j}(-1)^{i+j}\, \psi([\overline{L}_i, \overline{L}_j], \overline{L}_1, \ldots, \hat{\overline{L}}_i, \ldots, \hat{\overline{L}}_j, \ldots, \overline{L}_{q+1})$, for all $L_1, \ldots, L_{q+1} \in \Gamma(S)$, defines an element $\overline{\partial}_E^q\psi \in \mathcal{C}^{q+1}(X,E)$. Further $\overline{\partial}_E^{q+1}\overline{\partial}_E^q = 0$. The cohomology groups of the resulting complex $\{\mathcal{C}^q(X,E), \overline{\partial}_E^q\}$ are denoted by $H^q(X,E)$.

The CR structure S on X induces a natural filtration of the de Rham complex $\{\mathcal{A}^k(X), d\}$ with complex coefficients: Let $A^k(X) = \bigwedge^k \mathbb{C}T(X)^*$ and denote by $F^p(A^k(X))$ the subbundle of $A^k(X)$ consisting of all ϕ satisfying $\phi(Y_1, \ldots, Y_{p-1}, \overline{Z}_1, \ldots, \overline{Z}_{k-p+1}) = 0$ for all Y's in $\mathbb{C}T(X)$ and Z's in S. Then $A^k(X) = F^0(A^k(X)) \supset F^1(A^k(X)) \supset \cdots \supset F^k(A^k(X)) \supset F^{k+1}(A^k(X)) = 0$. Set $F^p(\mathcal{A}^k(X)) = \Gamma(F^p(A^k(X))$. Since $dF^p(\mathcal{A}^k(X)) \subset F^p(\mathcal{A}^{k+1}(X))$, the collection $\{F^p(\mathcal{A}^k(X))\}$ gives a filtration of the de Rham complex.

Consider the spectral sequence $\{E_r^{p,q}(X), d_r\}$ associated with the filtration $\{F^p(\mathcal{A}(X))\}$. Let

$$\begin{aligned} A^{p,q}(X) &= F^p(A^{p+q}(X)), & \mathcal{A}^{p,q}(X) &= \Gamma(A^{p,q}(X)), \\ C^{p,q}(X) &= A^{p,q}(X)/A^{p+1,q-1}(X), & \mathcal{C}^{p,q}(X) &= \Gamma(C^{p,q}(X)). \end{aligned}$$

First, $E_0^{p,q}(X) = \mathcal{C}^{p,q}(X)$ and $d_0 : \mathcal{C}^{p,q}(X) \longrightarrow \mathcal{C}^{p,q+1}(X)$ is the map induced by d. Note that $E_0^{k,0}(X) = \mathcal{C}^{k,0}(X) = \mathcal{A}^{k,0}(X) \subset \mathcal{A}^k(X)$.

Next, $E_1^{p,q}(X) = \dfrac{\operatorname{Ker}(d_0 : \mathcal{C}^{p,q}(X) \longrightarrow \mathcal{C}^{p,q+1}(X))}{\operatorname{Im}(d_0 : \mathcal{C}^{p,q-1}(X) \longrightarrow \mathcal{C}^{p,q}(X))}$ and $d_1 : E_1^{p,q}(X) \longrightarrow E_1^{p+1,q}(X)$ is the naturally induced map.

In particular,

$$\begin{aligned} E_1^{k,0}(X) &= \operatorname{Ker}(d_0 : \mathcal{C}^{k,0}(X) \longrightarrow \mathcal{C}^{k,1}(X)) \\ &= \{\varphi \in \mathcal{A}^{k,0}(X) : d\varphi \in \mathcal{A}^{k+1,0}(X)\} \end{aligned}$$

and d_1 is just d on $E_1^{k,0}(X) \subset \mathcal{A}^k(X)$.

$E_1^{k,0}(X)$ is called the space of holomorphic k-forms on X. Denoting $E_1^{k,0}(X)$ by $\mathcal{S}^k(X)$, we have the holomorphic de Rham complex $\{\mathcal{S}^k(X), d\}$.

Then,

$$\begin{aligned} E_2^{k,0}(X) &= \frac{\operatorname{Ker}(d : \mathcal{S}^k(X) \longrightarrow \mathcal{S}^{k+1}(X))}{\operatorname{Im}(d : \mathcal{S}^{k-1}(X) \longrightarrow \mathcal{S}^k(X))} \\ &= \frac{\{\text{closed holomorphic } k\text{-forms on } X\}}{\{\text{exact holomorphic } k\text{-forms on } X\}}. \end{aligned}$$

The groups $E_1^{p,q}(X)$ and $E_2^{k,0}(X)$ will be denoted by $H_{KR}^{p,q}(X)$ and $H_h^k(X)$ respectively. The former are the Kohn-Rossi cohomology groups and the latter are the holomorphic de Rham cohomology groups.

For computations, we need a more explicit description of $H_{KR}^{p,q}(X)$ and $H_h^k(X)$ in case X is a real hypersurface in a complex manifold M. In this case, the proof of Lemma 2.3 provides an isomorphism $\Phi : \hat{T}(X) \longrightarrow T^{1,0}(M)|_X$ of holomorphic vector bundles over X. For any holomorphic vector bundle E over X, $\bigwedge^p E^*$ is naturally a holomorphic vector bundle over X with respect to the following $\overline{\partial} : \Gamma(\bigwedge^p E^*) \longrightarrow \Gamma(\bigwedge^p E^* \otimes \overline{S}^*)$. Let $\theta \in \Gamma(\bigwedge^p E^*)$ and $L \in \Gamma(S)$. $(\overline{\partial}\theta)(\overline{L}) = \overline{L}\theta$ is defined by

$$(\overline{L}\theta)(u_1, \ldots, u_p) = \overline{L}(\theta(u_1, \ldots, u_p)) - \sum_{i=1}^{p} \theta(u_1, \ldots, \overline{L}u_i, \ldots, u_p)$$

for all $u_1, \ldots, u_p \in \Gamma(E)$. Let both $\bigwedge^p \hat{T}(X)^*$ and $\bigwedge^p T^{1,0}(M)|_X^*$ be made holomorphic vector bundles over X in this way. Then Φ induces an isomorphism $\Phi^* : \bigwedge^p T^{1,0}(M)|_X^* \longrightarrow \bigwedge^p \hat{T}(X)^*$ of holomorphic vector bundles over X, where $(\Phi^*\xi)(u_1, \ldots, u_p) = \xi(\Phi u_1, \ldots, \Phi u_p)$ for $\xi \in \bigwedge^p T^{1,0}(M)|_X^*$, $u_1, \ldots, u_p \in \hat{T}(X)$. It is useful to write down the $\overline{\partial}$ operator of $\bigwedge^p T^{1,0}(M)|_X^*$ in local holomorphic coordinates $z_1, \ldots, z_n$ of M. Any $\xi \in \Gamma(\bigwedge^p T^{1,0}(M)|_X^*)$ can be written as $\xi = \sum \xi_{i_1 \ldots i_p} dz^{i_1} \wedge \cdots \wedge dz^{i_p}$, $\xi_{i_1 \ldots i_p}$ being local functions on X. For any $L \in \Gamma(S)$, one checks that $\overline{L}\xi = \sum (\overline{L}\xi_{i_1 \ldots i_p}) dz^{i_1} \wedge \cdots \wedge dz^{i_p}$.

Lemma 2.4. *Let X be a real hypersurface in a complex manifold M.*

(1) *The two complexes* $\{\mathcal{C}^{p,q}(X), d_0\}$ *and* $\{\mathcal{C}^q(X, \bigwedge^p T^{1,0}M|_X^*), (-1)^p\overline{\partial}^q\}$ *can be naturally identified.*

(2) $\mathcal{S}^k(X)$ *can be identified with the subspace of* $\Gamma(\bigwedge^k T^{1,0}M|_X^*)$ *consisting of those* $\xi = \sum \xi_I dz^I$ *satisfying* $\overline{\partial}_b \xi_I = 0$ *for all* I.

Proof. First consider the map $\iota^p : A^{p,q}(X) \longrightarrow C^q(X, \bigwedge^p \hat{T}(X)^*) = \bigwedge^p \hat{T}(X)^* \otimes \bigwedge^q \overline{S}^*$ defined by $(\iota^p\phi)(\omega(Z_1), \ldots, \omega(Z_p); \overline{L}_1, \ldots, \overline{L}_q) = \phi(Z_1, \ldots, Z_p, \overline{L}_1, \ldots, \overline{L}_q)$ for all $\phi \in A^{p,q}(X)$, $\omega(Z_i) \in \hat{T}(X)$ where $Z_i \in \mathbb{C}T(X)$, and $L_j \in S$. ι^p is surjective, with kernel $A^{p+1,q-1}(X)$. Hence ι^p induces an isomorphism $\mathcal{C}^{p,q}(X) \longrightarrow \mathcal{C}^q(X, \bigwedge^p \hat{T}(X)^*)$ which we also denote by ι^p. A careful checking shows that for any $\varphi \in \mathcal{A}^{p,q}(X)$, $\iota^p d\varphi = (-1)^p\overline{\partial}^q \iota^p\varphi$. Thus $\{\mathcal{C}^{p,q}(X), d_0\}$ and $\{\mathcal{C}^q(X, \bigwedge^p \hat{T}(X)^*), (-1)^p\overline{\partial}^q\}$ can be naturally identified. Since $\bigwedge^p \hat{T}(X)^*$ and $\bigwedge^p T^{1,0}(M)|_X^*$ are isomorphic as holomorphic vector bundles over X, (1) follows. By (1), $\mathcal{S}^k(X) = \operatorname{Ker}(d_0 : \mathcal{C}^{k,0}(X) \longrightarrow \mathcal{C}^{k,1}(X))$ can be identified with $\operatorname{Ker}(\overline{\partial} : \mathcal{C}^0(X, \bigwedge^k T^{1,0}(M)|_X^*) \longrightarrow \mathcal{C}^1(X, \bigwedge^k T^{1,0}(M)|_X^*))$. Then (2) follows from the remark preceeding the lemma. q.e.d.

Definition 2.5. Let $L_1, \ldots, L_{n-1}$ be a local frame of the CR structure S on X, so that $\overline{L}_1, \ldots, \overline{L}_{n-1}$ is a local frame of $\overline{S}$. Choose a local section N of $\mathbb{C}T(X)$ such that $L_1, \ldots, L_{n-1}, \overline{L}_1, \ldots, \overline{L}_{n-1}, N$ span $\mathbb{C}T(X)$. Assuming that N is purely imaginary,

$$[L_i, \overline{L}_j] = \sum a_{ij}^k L_k + \sum b_{ij}^k \overline{L}_k + c_{ij}N$$

where (c_{ij}) is a Hermitian matrix called the Levi form of X. X is said to be strongly pseudoconvex if the Levi form is definite at each point of X. This condition is independent of the choice of $L_1, \ldots, L_{n-1}, N$.

Lemma 2.6. *Let X be a compact connected strongly pseudoconvex real hypersurface in a complex manifold M. If all $\varphi \in \mathcal{S}^k(X)$ can be extended to holomorphic k-forms on a common one-sided neighborhood U of X in M, then the complex $\{\mathcal{S}^k(X), d\}$ can be naturally identified with the complex $\{\Gamma(U, \Omega^k), d\}$, where $\Gamma(U, \Omega^k)$ denotes the space of holomorphic k-forms on $U \setminus X$ which extend smoothly (C^∞) up to the boundary component X.*

Proof. We have seen that any $\varphi \in \mathcal{S}^k(X)$ can be identified with some $\xi \in \Gamma(\bigwedge^k T^{1,0}(M)|_X^*)$ such that $\varphi(W_1, \ldots, W_k) = \xi(W_1^{1,0}, \ldots, W_k^{1,0})$

where $W_i \in \Gamma(\mathbb{C}TX)$ and $W_i^{1,0}$ denotes the (1,0) component of W_i in $\mathbb{C}TM$. Denote the holomorphic extension of ξ to U also by ξ. One defines $d\xi(W_1, \ldots, W_{k+1})$, $W_i \in \Gamma(\mathbb{C}TX)$, in the obvious way and checks that $d\varphi(W_1, \ldots, W_{k+1}) = d\xi(W_1, \ldots, W_{k+1})$, in particular when $W_{k+1} = N$, N as in Definition 2.5. q.e.d.

Finally, we have the following situation.

Theorem 2.7. *Let X be a compact connected $(2n-1)$-dimensional $(n \geqslant 2)$ strongly pseudoconvex* CR *manifold. Suppose that X is the boundary of a compact n-dimensional strongly pseudoconvex manifold M which is a modification of a compact Stein space V with only isolated singularities. Let A be the minimal compact analytic set in M which can be blown down to the isolated singularities. Then the complex $\{\mathcal{S}^k(X), d\}$ can be naturally identified with the complex $\{\Gamma(M \backslash A, \Omega^k), d\}$ (notation as in Lemma* 2.6*).*

Proof. One can take a one-convex exhaustion function ρ on the interior of M such that $\rho \geqslant 0$ everywhere and $\rho(y) = 0$ if and only if $y \in A$. Any $\varphi \in \mathcal{S}^k(X)$ can be extended to a one-sided neighborhood U of X in M, where U may be taken to be $M \setminus \{\rho \leqslant r\}$. By Andreotti and Grauert [1, Théorème 15], $\Gamma(M \setminus \{\rho \leqslant r\}, \Omega^k_{M \setminus \{\rho \leqslant r\}})$ is isomorphic to $\Gamma(M \setminus A, \Omega^k_{M \setminus A})$. The rest is as in Lemma 2.6. q.e.d.

3. Transversal holomorphic S^1-action and punctured holomorphic de Rham cohomology

In the following, we consider a compact connected strongly pseudoconvex CR manifold in $\mathbb{C}^N$ which admits a transversal holomorphic S^1-action. We recall:

Definition 3.1. Let X be a CR manifold as in Definition 2.1. A smooth S^1-action (of class C^1) on X is said to be holomorphic if it preserves the subspaces $\mathcal{H} \subset T(X)$ and commutes with J. It is said to be transversal if, in addition, the vector field v which generates the action, is transversal to $\mathcal{H}$ at all points of X.

Definition 3.2. Let φ_t be a continuous S^1-action on a complex analytic space V which preserves the regular points of V. The action is called weakly holomorphic (resp. holomorphic) if for all t, $\varphi_t^* \mathcal{O}_V^\omega = \mathcal{O}_V^\omega$ (resp. $\varphi_t^* \mathcal{O}_V = \mathcal{O}_V$), where $\mathcal{O}_V^\omega$ is the sheaf of germs of weakly holomorphic functions on V (i.e., the sheaf of germs of locally bounded holomorphic functions on the regular part of V).

Theorem 3.3 ([5]). *Let X be a compact connected* CR *manifold which bounds a compact complex analytic variety V of dimension n in $\mathbb{C}^N$. Then any transversal holomorphic S^1-action on X extends to a weakly holomorphic representation Φ of the analytic semigroup $\triangle^* = \{t \in \mathbb{C} : 0 < |t| \leqslant 1\}$ as a semigroup of analytic embeddings of V into itself. This action has a single fixed point x and given any neighborhood U of x in V, there is an $\varepsilon > 0$ so that $\Phi_t(V) \subset U$ for all $|t| < \varepsilon$.*

Indeed V has at most one singularity, namely at x. The following theorem and its proof is our key to compute the holomorphic de Rham cohomology $H_h^k(X)$.

Theorem 3.4. *Let X be a compact connected $(2n-1)$-dimensional $(n \geqslant 2)$ strongly pseudoconvex* CR *manifold with transversal holomorphic S^1-action φ_t. Suppose that X bounds a compact complex analytic variety V in $\mathbb{C}^N$ with x as its only singular point. Let U be any open neighborhood of x contained in the interior $\overset{\circ}{V}$ of V such that under the extended weakly holomorphic $\triangle^*$-action, $\Phi_t(U) \subset U$ for all $t \in \triangle^*$. Then the holomorphic de Rham cohomology groups of the complex manifolds $\overset{\circ}{V} \setminus \{x\}$ and $U \setminus \{x\}$ are naturally isomorphic. More precisely, the restriction map $r : H_h^k(\overset{\circ}{V} \setminus \{x\}) \longrightarrow H_h^k(U \setminus \{x\})$ is an isomorphism, $k \geqslant 0$.*

Proof. By Theorem 3.3, φ_t extends to a weakly holomorphic $\triangle^*$-action Φ_t and there exists $t_0 \in \triangle^*$ such that $\overset{\circ}{V} \supset U \supset \Phi_{t_0}(\overset{\circ}{V}) \supset \Phi_{t_0}(U)$. Consider the following natural maps induced by restrictions:

$$H_h^k(\overset{\circ}{V} \setminus \{x\}) \xrightarrow{r} H_h^k(U \setminus \{x\}) \xrightarrow{r_1} H_h^k(\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}) \xrightarrow{r_2} H_h^k(\Phi_{t_0}(U) \setminus \{x\}).$$

We first prove that the composition

$$H_h^k(\overset{\circ}{V} \setminus \{x\}) \xrightarrow{r_1 \circ r} H_h^k(\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\})$$

is equal to the map $H_h^k(\overset{\circ}{V} \setminus \{x\}) \xrightarrow{\Phi_{t_0}^{-1*}} H_h^k(\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\})$ induced by the inverse of the analytic embedding Φ_{t_0}. This can be done by means of a chain homotopy as follows. Let $\triangle_{t_0}^* = \{t \in \mathbb{C} : |t_0| \leqslant |t| \leqslant 1\}$. For each $t \in \triangle_{t_0}^*$, we have $\overset{\circ}{V} \setminus \{x\} \underset{\Phi_t^{-1}}{\overset{\Phi_t}{\rightleftarrows}} \Phi_t(\overset{\circ}{V}) \setminus \{x\} \supset \Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}$.

Write $\Psi_t = \Phi_t^{-1}|_{\Phi_{t_0}(\overset{\circ}{V})\setminus\{x\}}$ and factorize it by

$$\begin{array}{ccccc} \Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\} & \xrightarrow{\lambda_t} & (\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}) \times \triangle_{t_0}^* & \xrightarrow{\Psi} & \overset{\circ}{V} \setminus \{x\} \\ \\ z & \longmapsto & (z,t) & \longmapsto & \Psi_t(z). \end{array}$$

Thus $\Psi_t = \Psi \circ \lambda_t$, where Ψ is holomorphic in z and t. An arbitrary holomorphic k-form ω^* on $(\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}) \times \triangle_{t_0}^*$ may be written in local holomorphic coordinates $z_1, \ldots, z_n$ of $\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}$ as

$$\omega^* = \sum f_I(z,t) dz_I + \sum g_J(z,t) dt \wedge dz_J. \tag{3.1}$$

We have forms $\lambda_t^* \omega^* = \sum f_I(z,t) dz_I$ on $\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}$. Then on $(\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}) \times \triangle_{t_0}^*$,

$$\frac{d}{dt} \lambda_t^* \omega^* = \sum \frac{\partial f_I}{\partial t}(z,t) dz_I. \tag{3.2}$$

Also,

$$\begin{aligned} d\omega^* &= \sum \frac{\partial f_I}{\partial z_i}(z,t) dz_i \wedge dz_I + \sum \frac{\partial f_I}{\partial t}(z,t) dt \wedge dz_I \\ &\quad + \sum \frac{\partial g_J}{\partial z_i}(z,t) dz_i \wedge dt \wedge dz_J \end{aligned} \tag{3.3}$$

$$i\left(\frac{\partial}{\partial t}\right) d\omega^* = \sum \frac{\partial f_I}{\partial t}(z,t) dz_I - \sum \frac{\partial g_J}{\partial z_i}(z,t) dz_i \wedge dz_J$$

$$\lambda_t^*\left(i\left(\frac{\partial}{\partial t}\right) d\omega^*\right) = \sum \frac{\partial f_I}{\partial t}(z,t) dz_I - \sum \frac{\partial g_J}{\partial z_i}(z,t) dz_i \wedge dz_J.$$

On the other hand,

$$i\left(\frac{\partial}{\partial t}\right) \omega^* = \sum g_J(z,t) dz_J \tag{3.4}$$

$$d\left(i\left(\frac{\partial}{\partial t}\right) \omega^*\right) = \sum \frac{\partial g_J}{\partial z_i}(z,t) dz_i \wedge dz_J + \sum \frac{\partial g_J}{\partial t}(z,t) dt \wedge dz_J$$

$$\lambda_t^*\left(d\left(i\left(\frac{\partial}{\partial t}\right) \omega^*\right)\right) = \sum \frac{\partial g_J}{\partial z_i}(z,t) dz_i \wedge dz_J.$$

Treating (3.3) and (3.4) like (3.2) as equations on $\big(\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}\big) \times \triangle^*_{t_0}$, we get

$$\frac{d}{dt}\lambda_t^*\omega^* = \lambda_t^*\left(i\left(\frac{\partial}{\partial t}\right)d\omega^*\right) + d\left(\lambda_t^*\left(i\left(\frac{\partial}{\partial t}\right)\omega^*\right)\right). \tag{3.5}$$

For any holomorphic k-form ω on $\overset{\circ}{V} \setminus \{x\}$, putting $\omega^* = \Psi^*\omega$ in (3.5) gives

$$\frac{d}{dt}\Psi_t^*\omega = \left(\lambda_t^* i\left(\frac{\partial}{\partial t}\right)\Psi^*\right)d\omega + d\left(\lambda_t^* i\left(\frac{\partial}{\partial t}\right)\Psi^*\right)\omega. \tag{3.6}$$

Taking a smooth curve γ in $\triangle^*_{t_0}$ joining t_0 to 1 and integrating (3.6) along γ gives

$$\Psi_1^*\omega - \Psi_{t_0}^*\omega = Hd\omega + dH\omega \tag{3.7}$$

where $H\eta = \int_{t_0}^1 \lambda_t^* i\left(\frac{\partial}{\partial t}\right)\Psi^*\eta dt$ provides the desired chain homotopy. Observe that Ψ_1 is the inclusion map $\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\} \longrightarrow \overset{\circ}{V} \setminus \{x\}$ while Ψ_{t_0} is the biholomorphic map $\Phi_{t_0}^{-1} : \Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\} \longrightarrow \overset{\circ}{V} \setminus \{x\}$. Hence Ψ_1^* (respectively $\Psi_{t_0}^*$) induces $r_1 \circ r$ (respectively $\Phi_{t_0}^{-1^*}$) on the cohomology level. Then (3.7) implies that $r_1 \circ r = \Phi_{t_0}^{-1^*}$.

Now, since $\Phi_{t_0}^{-1}$ is a biholomorphic map, $\Phi_{t_0}^{-1*}$ is an isomorphism, hence so is $r_1 \circ r$. On the other hand, if U is a strongly pseudoconvex neighborhood of x in $\overset{\circ}{V}$ satisfying $U \supset \Phi_t(U)$ for all $t \in \triangle^*$, then we also have $U \setminus \{x\} \underset{\Phi_t^{-1}}{\overset{\Phi_t}{\rightleftarrows}} \Phi_t(U) \setminus \{x\} \supset \Phi_{t_0}(U) \setminus \{x\}$ for all $t \in \triangle^*_{t_0}$. By repeating the above argument, we see that

$$r_2 \circ r_1\big(= \Phi_{t_0}^{-1^*}\big) : H_h^k(U \setminus \{x\}) \longrightarrow H_h^k(\Phi_{t_0}(U) \setminus \{x\})$$

is an isomorphism. $r_1 \circ r$ and $r_2 \circ r_1$ bijective implies r_1 bijective. It follows that r is also bijective, hence an isomorphism. q.e.d.

Remark 3.5. For the simplest case where X is the unit sphere and V is the closed unit ball in $\mathbb{C}^2$, the S^1-action $\varphi_t(z_1, z_2) = (tz_1, tz_2)$ on X extends to the $\triangle^*$-action $\Phi_t(z_1, z_2) = (tz_1, tz_2)$ on V with fixed point at the origin 0. In this case, if $r : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ is C^∞ and strictly increasing in each variable, and if $r(|z_1|,|z_2|)<0$ defines an open neighborhood U of 0 contained in the open unit ball, then $\Phi_t(U)\subset U$ for all $t\in\triangle^*$. For example, for $0<a_1,a_2<1$, $\dfrac{|z_1|^2}{a_1^2}+\dfrac{|z_2|^2}{a_2^2}<1$ defines such a U which is biholomorphically equivalent to the open unit ball.

On the other hand, there are strongly pseudoconvex neighborhoods U_1 of 0 in V for which the condition for all $t\in\triangle^*$, $\Phi_t(U_1)\subset U_1$ fails. For example, let U_1 be defined by $|z_1|^2+|z_2|^2+\lambda(z_1+\overline{z}_1)<\frac{1}{2}$, where $\lambda\in\mathbb{R}$. If $0<\lambda<\frac{1}{6}$, one checks that:

(1) U_1 is a strongly pseudoconvex domain contained in the open unit ball.

(2) For $(z_1,z_2)\in\partial U_1$, $z_1=x_1+iy_1$ and $t=\cos\theta+i\sin\theta$, $x_1<0$ and $y_1\sin\theta<0$ implies that $(tz_1,tz_2)\notin\overline{U_1}$.

A slight variation of the above proof yields the following corollary.

Corollary 3.6. *Let X and (V,x) be as in Theorem 3.4. Denote by $\Gamma(\overset{\circ}{V}\setminus\{x\},\Omega^k)$ the space of holomorphic k-forms on $\overset{\circ}{V}\setminus\{x\}$ and by $\Gamma(V\setminus\{x\},\Omega^k)$ the subspace consisting of those holomorphic k-forms that extend smoothly (C^∞) up to the boundary component X. Then the two complexes $\{\Gamma(\overset{\circ}{V}\setminus\{x\},\Omega^\bullet),d\}$ and $\{\Gamma(V\setminus\{x\},\Omega^\bullet),d\}$ have isomorphic cohomology groups $H_h^k(\overset{\circ}{V}\setminus\{x\})$ and $H_h^k(V\setminus\{x\})$ respectively.*

Proof. Consider the following chain maps

$$\Gamma(V\setminus\{x\},\Omega^\bullet)\xrightarrow{r}\Gamma(\overset{\circ}{V}\setminus\{x\},\Omega^\bullet)\xrightarrow{r_1}$$
$$\Gamma(\Phi_{t_0}(V)\setminus\{x\},\Omega^\bullet)\xrightarrow{r_2}\Gamma(\Phi_{t_0}(\overset{\circ}{V})\setminus\{x\},\Omega^\bullet)$$

for any $|t_0|<1$. Then we have the induced maps

$$H_h^k(V\setminus\{x\})\xrightarrow{r}H_h^k(\overset{\circ}{V}\setminus\{x\})\xrightarrow{r_1}$$
$$H_h^k(\Phi_{t_0}(V)\setminus\{x\})\xrightarrow{r_2}H_h^k(\Phi_{t_0}(\overset{\circ}{V})\setminus\{x\}).$$

To prove $r_1\circ r=\Phi_{t_0}^{-1*}$, we consider $V\setminus\{x\}\ \underset{\Phi_t^{-1}}{\overset{\Phi_t}{\rightleftarrows}}\ \Phi_t(\overset{\circ}{V})\setminus\{x\}$ $\supset\Phi_{t_0}(\overset{\circ}{V})\setminus\{x\}$ for all $t\in\triangle_{t_0}^*$. For any $\omega\in\Gamma(V\setminus\{x\},\Omega^k)$, $\Psi_1^*\omega-$

$\Psi_{t_0}^*\omega = Hd\omega + dH\omega$ holds with $H\omega \in \Gamma(V \setminus \{x\}, \Omega^k)$. It follows that $r_1 \circ r = \Phi_{t_0}^{-1*}$, hence is an isomorphism.

The same argument for $\overset{\circ}{V}$ in place of V shows that $r_2 \circ r_1$ is an isomorphism. As before, we conclude that r is an isomorphism. q.e.d.

It is now natural to relate to the theory of punctured local holomorphic de Rham cohomology developed in [3].

Definition 3.7. Let V be a complex analytic space with only isolated singularities. For any $x \in V$, the punctured local holomorphic de Rham cohomology $H_h^k(V,x)$ is defined to be $\varinjlim H_h^k(U \setminus \{x\})$, where U runs over strongly pseudoconvex open neighborhoods of x in V.

We are now ready to prove Theorem A.

Proof of Theorem A. By Theorem 2.7, the complexes $\{\mathcal{S}^\bullet(X), d\}$ and $\{\Gamma(M \setminus A, \Omega^\bullet), d\}$ are isomorphic. The latter is clearly isomorphic to $\{\Gamma(V \setminus \{x\}, \Omega^\bullet), d\}$. Hence $H_h^k(X)$ is isomorphic to $H_h^k(V \setminus \{x\})$. By Corollary 3.6, $H_h^k(X)$ is isomorphic to $H_h^k(\overset{\circ}{V} \setminus \{x\})$. It suffices to check that $\varinjlim H_h^k(U \setminus \{x\}) = H_h^k(\overset{\circ}{V} \setminus \{x\})$.

We shall define, for any open neighborhood U of x in $\overset{\circ}{V}$ satisfying the condition in Theorem 3.4, a homomorphism $m_U : H_h^k(U \setminus \{x\}) \longrightarrow H_h^k(\overset{\circ}{V} \setminus \{x\})$, such that the following conditions hold:

(1) For any open neighborhoods $U \supset W$ of x in $\overset{\circ}{V}$, if

$$r_{WU} : H_h^k(U \setminus \{x\}) \longrightarrow H_h^k(W \setminus \{x\})$$

is the map induced by restriction, then $m_W r_{WU} = m_U$.

(2) If there are homomorphisms $n_U : H_h^k(U \setminus \{x\}) \longrightarrow N$ into some group N satisfying $n_W r_{WU} = n_U$ for all open neighborhoods $U \supset W$ of x in $\overset{\circ}{V}$, then there is a unique homomorphism

$$n : H_h^k(\overset{\circ}{V} \setminus \{x\}) \longrightarrow N$$

such that $n \circ m_U = n_U$.

For any open neighborhood U of x in $\overset{\circ}{V}$, take $t_0 \in \triangle^*$ such that $U \supset \Phi_{t_0}(\overset{\circ}{V})$ and set $m_U = \Phi_{t_0}^* r_{\Phi_{t_0}(\overset{\circ}{V}), U}$. To check that m_U is independent of the choice of t_0, observe that if $|t_1| \leqslant |t_0|$, then by the proof of Theorem 3.4, $\Phi_{\frac{t_1}{t_0}}^{-1*} = r : H_h^k(\Phi_{t_0}(\overset{\circ}{V}) \setminus \{x\}) \longrightarrow H_h^k(\Phi_{t_1}(\overset{\circ}{V}) \setminus \{x\})$, hence $\Phi_{t_0}^* r_{\Phi_{t_0}(\overset{\circ}{V}),U} = \Phi_{t_1}^* \Phi_{\frac{t_1}{t_0}}^{-1*} r_{\Phi_{t_0}(\overset{\circ}{V}),U} = \Phi_{t_1}^* r_{\Phi_{t_1}(\overset{\circ}{V}),U}$.

To check (1), take $t \in \triangle^*$ such that $U \supset W \supset \Phi_t(\overset{\circ}{V})$. Then $m_W r_{WU} = \Phi_t^* r_{\Phi_t(\overset{\circ}{V}),W} r_{WU} = m_U$. To check (2), take $n = n_{\overset{\circ}{V}}$. Then $nm_U = n_{\overset{\circ}{V}} \Phi_t^* r_{\Phi_t(\overset{\circ}{V}),U}$, where $t \in \triangle^*$ and $U \supset \Phi_t(\overset{\circ}{V})$. By Theorem 3.4, $\Phi_t^{-1*} = r_{\Phi_t(\overset{\circ}{V}),\overset{\circ}{V}}$, hence the given condition $n_{\Phi_t(\overset{\circ}{V})} r_{\Phi_t(\overset{\circ}{V}),\overset{\circ}{V}} = n_{\overset{\circ}{V}}$ may be written as $n_{\Phi_t(\overset{\circ}{V})} = n_{\overset{\circ}{V}} \Phi_t^*$. Then $nm_U = n_{\Phi_t(\overset{\circ}{V})} r_{\Phi_t(\overset{\circ}{V}),U} = n_U$. The uniqueness of n is clear from the requirement $nm_{\overset{\circ}{V}} = n_{\overset{\circ}{V}}$. q.e.d.

To prove Theorem B, we use the following theorem on the punctured local holomorphic de Rham cohomology in the hypersurface case.

Theorem 3.8 ([3]). *Let $(V,0) = \{(z_0,\dots,z_n) \in \mathbb{C}^{n+1} : f(z_0,\dots,z_n) = 0\}$ be a hypersurface with the origin as an isolated singular point. Then:*

(1) $\dim H_h^k(V,0) = 0$ *for* $1 \leqslant k \leqslant n-2$.

(2) $\dim H_h^n(V,0) - \dim H_h^{n-1}(V,0) = \mu - \tau$, *where*

$$\mu = \dim \mathbb{C}\{z_0,\dots,z_n\} \Big/ \left(\frac{\partial f}{\partial z_0},\dots,\frac{\partial f}{\partial z_n}\right)$$

is the Milnor number and

$$\tau = \dim \mathbb{C}\{z_0,\dots,z_n\} \Big/ \left(f, \frac{\partial f}{\partial z_0},\dots,\frac{\partial f}{\partial z_n}\right)$$

is the Tjurina number of the singularity $(V,0)$ respectively.

In particular, $\dim H_h^n(V,0) = \dim H_h^{n-1}(V,0)$ *if and only if $(V,0)$ is a quasi-homogeneous singularity, i.e., f is a weighted homogeneous polynomial after a holomorphic change of coordinates.*

Proof of Theorem B. In view of Corollary 2.7 of [5] we know that the hypersurface singularity (V,x) is quasi-homogeneous. Then Theorem B follows from Theorem A and Theorem 3.8. q.e.d.

Let $X \subset \mathbb{C}^N$ be a compact connected strongly pseudoconvex 3-dimensional CR manifold with a transversal holomorphic S^1-action. In view of Theorem 2.1 of [5], there exists a holomorphic equivariant embedding $X \hookrightarrow \widetilde{V}$ as a real hypersurface in a 2-dimensional algebraic variety $\widetilde{V} \subset \mathbb{C}^{\widetilde{N}}$ with a linear $\mathbb{C}^*$-action; moreover, the linear $\mathbb{C}^*$-action has exactly one fixed point x, which is the only possible singularity of $\widetilde{V}$. We assume here that the fixed point x is a rational singularity. By Theorem A, $H_h^k(X) = H_h^k(V, x)$ where V is the compact subset of $\widetilde{V}$ bounded by X.

We now recall the following important result of Campana and Flenner.

Proposition 3.9 ([2]). *If $(V, 0)$ is rational isolated singularity of dimension $n \geqslant 2$, then any closed holomorphic p-form η on $V \setminus \{0\}$ with $1 \leqslant p \leqslant 2$ is exact, i.e., after shrinking V as a neighborhood of 0, there exists a $(p-1)$-form ξ on $V \setminus \{0\}$ with $d(\xi) = \eta$.*

Proof of Theorem C. By Theorem A, $H_h^k(X) \cong H_h^k(V, x)$, for $k = 1$, 2. The above proposition of Campana and Flenner says that $H_h^k(V, x) = 0$, $k = 1, 2$. q.e.d.

Proof of Theorem D. Let x be the singularity of V. Then (V, x) is an isolated quasi-homogeneous hypersurface singularity. If $H_h^2(X) = 0$, then Theorem A of [6] implies that $\dim H_h^2(M) = 0 = s$, where $s = \dim \Gamma(M \setminus A, \Omega^2)/(d\Gamma(M \setminus A, \Omega^1) + \Gamma(M, \Omega^2))$ is an invariant of (V, x). Vanishing of s implies $H^1(A, \mathbb{C}) = 0$ by Theorem B of [6].

Conversely if $H^1(A, \mathbb{C}) = 0$, then Theorem B of [6] implies $s = 0$ and hence $H_h^1(M) = 0 = H_h^2(M)$. In view of Theorem A of [6], we have $H_h^1(X) \cong H_h^1(M) = 0$ and $\dim H_h^2(X) = \dim H_h^2(M) + s = 0$. q.e.d.

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