Stable Matching (Gale & Shapley '62)

Given a set of preferences among men/women.

We want to find a matching (# of men = # of women)

Such that: \( f(m,w) \) is NOT matched:

at least one of the following is true:

1. \( m \) prefers his current partner over \( w \).
2. \( w \) prefers his current partner over \( m \).

\( \Rightarrow \) So they cannot deviate.

We call this a "stable Matching".

Example 1: \( m \) likes \( w \) over \( w' \) \( = \) stable!

\( \begin{array}{c}
\text{Example 2: } m \text{ prefers } w \text{ over } w' \\
\text{m' prefers } w' \text{ over } w \end{array} \) 

\( \begin{array}{c}
\text{w prefers } m \text{ over } m' \\
\text{w' prefers } m \text{ over } m' \end{array} \) 

\( \Rightarrow \) stable

\( \times \) not stable!

\( (m,w) \) is not matched, but both prefer each other.

Example 3: \( m \) prefers \( w > w' \)

\( m' \) prefers \( w' > w \)

\( w \) prefers \( m' > m \)

\( w' \) prefers \( m > m' \)

\( \Rightarrow \) It's possible to have multiple stable matchings!