Outline

Show

- Vertex 3-Coloring
- Hamiltonian Cycle
- Super Mario

are NP-Hard.
Vertex 3-Coloring

Input: a graph

Output: color each vertex using 1 of the 3 colors, so that adjacent vertices do not get the same color.
3-Coloring:
3-Coloring: Yes instance
3-Coloring: No instance
3SAT \leq_p 3\text{-Coloring}

Satisfiable formula \iff

Unsatisfiable formula \iff
Vertex 3-Coloring

[Garey, Johnson, Stockmeyer 1976]
Vertex 3-Coloring

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3SAT \leq_p 3\text{-Coloring}

• Consequence:

3\text{-Coloring is NP-Complete.}
(Because 3\text{-Coloring is also in NP.)}
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Hamiltonian Cycle
Hamiltonian Cycle

• Input: a (directed) graph.

• Solution: a cycle visiting every vertex exactly once.
Variable Gadget

Direction we travel along this chain represents whether to set the variable to true or false.
Clause Gadget

Add a new node for each clause:

$C_k$

Connect it this way if $\overline{x_j}$ in $C_k$

$C_j$

Connect it this way if $x_j$ in $C_k$

$x_i$

...
3SAT $\leq_p$ Hamiltonian Cycle
3SAT $\leq_p$ Hamiltonian Cycle

3SAT $\leq_p$ Hamiltonian Path

Dec 5, 2017  Yu Cheng
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Fun with Hardness Proofs

Algorithmic Lower Bounds:
Fun with Hardness Proofs

Erik Demaine

http://courses.csail.mit.edu/6.890/fall14/lectures/
Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo 2012]
Super Mario Bros.
Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo, Viglietta 2014]

\[(x \lor \neg y \lor z) \land (x \lor y \lor \neg y) \land (\neg x \lor \neg y \lor \neg z) \land (\neg x \lor \neg y \lor \neg z)\]
Super Mario Bros. is NP-Hard
[Aloupis, Demaine, Guo, Viglietta 2014]