FROM 7.5:

2.

$$
\begin{aligned}
(a+4b)^5 &= \sum_{k=0}^{5} \binom{5}{k} a^{5-k}(4b)^k \\
&= \binom{5}{0} a^5(4b)^0 + \binom{5}{1} a^4(4b)^1 + \binom{5}{2} a^3(4b)^2 + \binom{5}{3} a^2(4b)^3 \\
&\quad + \binom{5}{4} a^1(4b)^4 + \binom{5}{5} a^0(4b)^5 \\
&= a^5 + 20a^4b + 160a^3b^2 + 640a^2b^3 + 1280ab^5 + 1024b^5
\end{aligned}
$$

12.

$$
\begin{aligned}
\left(2x - \frac{3}{x^2}\right)^{58} &= \sum_{k=0}^{58} \binom{58}{k} (2x)^{58-k} \left(-\frac{3}{x^2}\right)^k \\
&= \sum_{k=0}^{58} \binom{58}{k} 2^{58-k} x^{58-k} (-3)^k \left(x^{-2}\right)^k \\
&= \sum_{k=0}^{58} \binom{58}{k} 2^{58-k} (-3)^k x^{58-3k}
\end{aligned}
$$

The coefficient of $x^2 5$ is the coefficient of the term with $58 - 3k = 25$, so $k = 11$. This coefficient is $\binom{58}{11} 2^{58-11}(-3)^{11} = \binom{58}{11} 2^{47}(-3)^{11}$.

21. (a) $1 + 2\binom{4}{1} + 4\binom{4}{2} + 8\binom{4}{3} + 16 = 1 + 8 + 24 + 32 + 16 = 81 = 3^4$.
(b)

$$
\begin{aligned}
\binom{n}{0} + 2\binom{n}{1} + 4\binom{n}{2} + \cdots + 2^n\binom{n}{n} &= \sum_{k=0}^{n} \binom{n}{k} 2^k \\
&= \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} 2^k \\
&= (1+2)^n = 3^n
\end{aligned}
$$

FROM 8.1:

9. *Step 1.* Set $T = 0$.
Step 2. For i from 1 to n, if $a_i = x$, then replace T by $T + 1$.
Step 3. Output T.

FROM 8.2:

5. In algorithm $\mathcal{A}$, there is one multiplication in each reiteration of the for loop, which is executed 2^n times. The complexity function is $f(n) = 2^n$. In algorithm $\mathcal{B}$, there is again one multiplication in each iteration of the for loop, which is executed n times. The complexity function is $g(n) = n$. Algorithm $\mathcal{B}$ is more efficient because $n \prec 2^n$. Proof of this is by induction on n. We show that $n < 2^n$ for $n \geq 1$. Base case: $n = 1$. Clearly, $1 < 2^1 = 2$ so it is true for $n = 1$. Inductive step: Assume $n < 2^n$ and prove $n+1 < 2^{n+1}$. $2^{n+1} = 2 \cdot 2^n > 2 \cdot n \geq n+1$. We thus have $n < 2^n$ for $n \geq 1$ so take $c = 1$ and and $n_0 = 1$ to get $n \prec 2^n$.

7. (b) For $n \geq 1$, $17n^4+8n^3+5n^2+6n+1 \leq 17n^4+8n^4+5n^4+6n^4+n^4 = 37n^4$. So if we take $c = 37$ and $n_0 = 1$, we have $17n^4 + 8n^3 + 5n^2 + 6n + 1 \prec n^4$. (d) For $n \geq 1$, $2^n \leq 3^n$, so taking $c = 1$ and $n_0 = 1$, we have $2^n \prec 3^n$.

8. (d) $f \prec g$ but $f \not\asymp g$. Suppose $g \prec f$. Then there exists constants c and n_0 such that $3^n \leq c2^n$ for all $n \geq n_0$. Then $c \geq \frac{3^n}{2^n} = \left(\frac{3}{2}\right)^n$. But this can't be true because c is a constant but $\left(\frac{3}{2}\right)^n$ grows exponentially. Thus $g \not\prec f$.

9. We know that $f = \mathcal{O}(h)$, so there are constants c_1 and n_1 such that $f(n) \leq c_1h(n)$ for all $n \geq n_1$. We know that $g = \mathcal{O}(h)$, so there are constants c_2 and n_2 such that $f(n) \leq c_2h(n)$ for all $n \geq n_2$. Set $c = c_1 + c_2$ and $n_0 = \max(n_1, n_2)$. Note that $n_0 \geq n_1$ and $n_0 \geq n_2$, so for all $n \geq n_0$, $f(n) + g(n) \leq c_1h(n) + c_2h(n) = (c_1 + c_2)h(n) = ch(n)$, so $f + g = \mathcal{O}(h)$.