Spring 2003 Name _____

MCS 261:01 Hour Exam 1

1. Two tables of rules are provided on the back of this page.
2. Write you solutions in your exam book.
3. Turn in this sheet along with your exam book.
4. Show and explain all your work. An unjustified answer may not receive credits.

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1. Writ down the negation of following statements (5 points each).
   (a) Every integer is divisible by a prime.
   (b) For any real number $x > 0$, there is an integer n such that $n > x$.

2. Define $f : \mathbf{N} \to \mathbf{N}$ by $f(x) = x^2 + x$. Determine with reasons whether or not f is one-to-one or onto. (20 points)

3. Define $\sim$ on an interval $[-1, 2)$ by $a \sim b$ if and only if $\lfloor a \rfloor = \lfloor b \rfloor$.
   1. Show that $\sim$ defines an equivalence on $[-1, 2)$. (10 points)
   2 Find all equivalence classes. (10 points)

4. (a) Simplify $(p \vee q) \vee [(q \vee \neg r) \wedge (p \vee r)]$. (15 points)
   (b) Determine whether the following is a valid logical argument. (15 points)

   If I wear a purple coat and don't wear blue shoes, then I wear red socks.
   I am wearing a purple coat.
   If I wear blue shoes or red socks, then I wear a green hat.

   I am wearing a green hat.

5. Suppose that $A \subset B$ and $C \subset B^c$. Show that $A \cap C = \emptyset$. (20 points)

Some Basic Logical Equivalences

1. Idempotence
   (a) $(p \vee p) \iff p$
   (b) $(p \wedge p) \iff p$
2. Commutativity
   (a) $(p \vee q) \iff (q \vee p)$
   (b) $(p \wedge q) \iff (q \wedge p)$
3. Associativity
   (a) $((p \vee q) \vee r) \iff (p \vee (q \vee r))$
   (b) $((p \wedge q) \wedge r) \iff (p \wedge (q \wedge r))$
4. Distributivity
   (a) $(p \vee (q \wedge r)) \iff ((p \vee q) \wedge (p \vee r))$
   (b) $(p \wedge (q \vee r)) \iff ((p \wedge q) \vee (p \wedge r))$
5. Double Negation $\neg(\neg p) \iff p$
6. De Morgan's Laws
   (a) $\neg(p \vee q) \iff ((\neg p) \wedge (\neg q))$
   (b) $\neg(p \wedge q) \iff ((\neg p) \vee (\neg q))$
7. (a) $(p \vee \mathbf{1}) \iff \mathbf{1}$
   (b) $(p \wedge \mathbf{1}) \iff p$
8. (a) $(p \vee \mathbf{0}) \iff p$
   (b) $(p \wedge \mathbf{0}) \iff \mathbf{0}$
9. (a) $(p \vee (\neg p)) \iff \mathbf{1}$
   (b) $(p \wedge (\neg p)) \iff \mathbf{0}$
10. (a) $\neg\mathbf{1} \iff \mathbf{0}$
    (b) $\neg\mathbf{0} \iff \mathbf{1}$
11. $(p \to q) \iff [(\neg q) \to (\neg p)]$
12. $(p \leftrightarrow q) \iff [(p \to q) \wedge (q \to p)]$
13. $(p \to q) \iff [(\neg p) \vee q]$

Some Valid Logical Arguments

- Modus ponens $\dfrac{\begin{array}{c} p \\ p \to q \end{array}}{q}$
- Modus tollens $\dfrac{\begin{array}{c} p \to q \\ \neg q \end{array}}{\neg p}$
- Disjunctive syllogism $\dfrac{\begin{array}{c} p \vee q \\ \neg p \end{array}}{q}$
- Chain rule $\dfrac{\begin{array}{c} p \to q \\ q \to p \end{array}}{p \to r}$
- Resolution $\dfrac{\begin{array}{c} p \vee r \\ q \vee \neg r \end{array}}{p \vee q}$