

THE SOJOURN TIME DISTRIBUTION IN PROCESSOR-SHARING QUEUES

One of the main measures of performance in a processor sharing (PS) queueing system is a tagged customer's sojourn time distribution, conditioned on that customer's service time. In Chapters 2 and 3 we consider the $M/M/1$ -PS queue and $M/G/1$ -PS queue respectively, and obtain the conditional sojourn time distribution in various asymptotic limits. These include large time and/or large service request, and heavy traffic, where the arrival rate is only slightly less than the service rate. In the $M/M/1$ -PS queue, our results easily reproduce the unconditional asymptotics in Pollaczek (1946), Flatto (1997) and Morrison (1985), and lead to a better understanding of what ranges of service time are important for removing the condition. In the $M/G/1$ -PS queue, we assume that the service density decays at least exponentially fast. Our results demonstrate the possible tail behaviors of the unconditional distribution, which was previously known in the cases $G = M$ and $G = D$ (where it is purely exponential).

For PS models one is also interested in the sojourn time conditioned on the number of other customers in the system when the tagged customer arrives. In Chapter 4 the $M/M/1$ -PS queue is studied, and a new exact representation is obtained for the conditional sojourn time distribution, using a discrete Green's function. This is shown to be equivalent to the classic result of Pollaczek (1946). Then various asymptotic limits are obtained, including large time and/or large number of customers present, and heavy traffic. From these results all of the expansions in Pollaczek (1946), Flatto (1997) and Morrison (1985) can again be recovered.