Binomial Trials

Binomial trials are a way of modeling a specific family of experiments. In these a certain trial is conducted \(n\) times, each time with the same probability \(p\) of success. The observed data value is the number of successes. Often we say \(q = 1 - p\) is the probability of failure. To compute the probability of exactly \(k\) successes, we use the following formula:

\[
Pr(X = k) = \binom{n}{k} p^k q^{n-k}
\]

We say that \(X\) is a binomial random variable with \(n\) and \(p\) as parameters.

To compute \(Pr(X \leq k)\) we must add up the cases \(Pr(X = 0) + Pr(X = 1) + \cdots + Pr(X = k)\).

**ex)** A die is rolled 9 times. What is the probability of rolling only one 6?

Because the probability of rolling a 6 is \(\frac{1}{6}\) and we are repeating the roll 9 times, this is a binomial trial. Using the binomial trials formula with \(n = 9\), \(p = \frac{1}{6}\), and \(k = 1\), we have

\[
Pr(X = 1) = \binom{9}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^8 \approx 0.3489.
\]

**ex)** A basketball player makes free-throws 80% of the time. What is the probability he makes exactly 3 out of 5 shots?

In this example, \(n=5\), \(p=.80\) and we want to know \(Pr(X = 3)\). This is \(\binom{5}{3} \cdot .80^3 \cdot .20^2 \approx .2048\)

**ex)** 19\$ of the population uses a certain band of detergent. From a sample of 15 shoppers, what is the probability that more than 2 use this brand?

To calculate \(Pr(X > 2)\) we would have to add up all cases \(Pr(X = 3) + Pr(X = 4) + \cdots + Pr(X = 15)\) which would be a tedious calculation. Instead we can use the complement rule: The probability more than 2 use the brand is 1 minus the probability that less than 3 use the brand; i.e. \(Pr(X > 2) = 1 - Pr(X = 0\text{ or }X = 1\text{ or }X = 2) = 1 - \left(\binom{15}{0}\cdot .19^0\cdot .81^{15} + \binom{15}{1}\cdot .19^1\cdot .91^{14} + \binom{15}{2}\cdot .19^2\cdot .81^{13}\right) \approx .5635\)

**ex)** For a 5 day vacation in Aspen with a 30% chance of snow each day, create a probability distribution for all possible numbers of snowy days.

<table>
<thead>
<tr>
<th>(k): # Snow Days</th>
<th>(Pr(X = k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\binom{5}{0}\cdot .3^0\cdot .7^5 \approx .168)</td>
</tr>
<tr>
<td>1</td>
<td>(\binom{5}{1}\cdot .3^1\cdot .7^4 \approx .360)</td>
</tr>
<tr>
<td>2</td>
<td>(\binom{5}{2}\cdot .3^2\cdot .7^3 \approx .309)</td>
</tr>
<tr>
<td>3</td>
<td>(\binom{5}{3}\cdot .3^3\cdot .7^2 \approx .132)</td>
</tr>
<tr>
<td>4</td>
<td>(\binom{5}{4}\cdot .3^4\cdot .7^1 \approx .028)</td>
</tr>
<tr>
<td>5</td>
<td>(\binom{5}{5}\cdot .3^5\cdot .7^0 \approx .002)</td>
</tr>
</tbody>
</table>

What is the probability of getting 3 or more snow days?

We can add up the probabilities from the distribution: \(.132 + .028 + .002 = .162\)