The Normal distribution is a continuous probability distribution, as opposed to a discrete distribution. A continuous random variable can take any real value along an interval, while a discrete random variable can only take specific values.

ex) The exact height of a random UIC student is a continuous random variable, while the shoe size is discrete.

The Normal distribution is a very common distribution and incredibly useful in statistics. Although we cannot draw a histogram for a continuous probability distribution, we can illustrate its probability density function which is analogous.

This is a typical normal curve. The normal curve is entirely characterized by two parameters: mean and standard deviation. The mean gives the center of the curve (where its peak is) and the standard deviation describes how flat/wide or narrow/tall the curve is. We often will use the notation \(X \sim N(\mu, \sigma)\) to mean “\(X\) is a normal random variable with mean \(\mu\) and standard deviation \(\sigma\)”.

A Standard Normal has mean \(\mu=0\) and \(\sigma=1\). We use \(Z\) to represent a standard normal random variable. We can always standardize or un-standardize using the following formulas:

\[
 z = \frac{x - \mu}{\sigma} \quad \text{or} \quad x = (z \cdot \sigma) + \mu
\]

For the area under the curve between two values is the probability that the random variable is within that interval. So if I want to know \(P(X<5)\)
Where \( X \sim N(4,2) \) this is the same as  
\[
P(Z < \frac{5-4}{2}) = P(Z < .5) = .6915
\]
You can find the probability of a standard normal from a Z-Table.

**ex) What is \( P(-1 < Z < .5) \)?**

The Z-table gives us the area to the left of .5 is .6915, and the area to the left of -1 is .1587. Thus the area between them is .6915-.1587=.5328

**ex) If \( X \) is normal with mean 5 and std dev 2, what is \( P(4 < X < 8) \)?**

We can standardize 4 and 8: 
\[
Pr(4 < X < 8) = Pr(\frac{4-5}{2} < Z < \frac{8-5}{2}) = Pr(-.5 < Z < 1.5) = Pr(Z < 1.5) - Pr(Z < -.5) \approx .6247
\]

**Percentiles**
If a value \( S \) is the \( p \)th percentile, that means \( p\% \) of the data falls below \( S \), \((100-p)\% \) is above \( S \). In other words, the probability of being less than \( S \) is \( p\% \).

**ex) \( X \) is normal with mean 5 and std dev 2, what is the 83rd percentile?**

We want to find \( x \) such that  
\[
Pr(X < x) = .83
\]
Thus  
\[
Pr(Z < \frac{x-5}{2}) = .83
\]
From the z-table, we find that \( z = .9542 \) is such that  
\[
Pr(Z < .9542) = .83
\]
so  
\[
\frac{x-5}{2} = .9542
\]
so \( x = 6.9084 \).