3.1 Sample Spaces

Definition 3.1. An experiment is any process that generates a set of data.

Definition 3.2. The set of all possible outcomes of a statistical experiment is called the sample space and is represented by the symbol $S$. Each outcome of the experiment is an element of the sample space.

Example 3.3. Suppose you are to flip a fair coin twice, and record the flips. The sample space would be

$$S = \{HH, HT, TH, TT\}.$$  

Example 3.4. Suppose you flip a fair coin twice. If both flips are heads, then you roll a 6-sided die. The sample space may be represented by the set

$$S = \{1, 2, 3, 4, 5, 6, HT, TH, TT\}.$$  

Sample spaces use set notation, so to describe complicated sample spaces, we may use the rule method.

Example 3.5. Suppose we are to ask a person how many birthday parties they have attended. Then the sample space may be represented as

$$S = \{x|x \in \mathbb{Z} \text{ and } x \geq 0\}.$$  

3.1.1 Events and Set Operations

Definition 3.6. An event is a subset of a sample space.

Example 3.7. Suppose in an experiment we are to draw a random card from a standard deck of 52 cards. The sample space is

$$S = \{2\spadesuit, \ldots, A\spadesuit, \ldots, 2\clubsuit, \ldots, A\clubsuit\}.$$  

Some events are:

$$A = \{x|x \text{ is a heart}\}$$  
$$B = \{x|x \text{ is a face card}\}$$  

The empty set, or $\emptyset$ is used to represent an event that cannot happen.

Definition 3.8. The complement of an event $A$ is the set of all outcomes of $S$ that do not belong to $A$, and is represented by $A'$

Definition 3.9. The union of two events $A$ and $B$, represented by $A \cup B$ is the set of all outcomes in either $A$ or $B$ (or both).
Definition 3.10. The **intersection** of two events $A$ and $B$, represented by $A \cap B$ is the set of all outcomes common to both $A$ and $B$. If $A \cap B = \emptyset$ then we say $A$ and $B$ are **mutually exclusive** (or **disjoint**).

Example 3.11. Suppose you are to roll a 10 sided die once and observe the number rolled. Let $A =$ “The number rolled is even” and $B =$ “The number rolled is divisible by 3”. We could instead write

$$A = \{2, 4, 6, 8, 10\}, B = \{3, 6, 9\}.$$ 

The complement events are

$$A' = \{1, 3, 5, 7, 9\}, B' = \{1, 2, 4, 5, 7, 8, 10\}.$$ 

The union and intersection respectively are

$$A \cup B = \{2, 3, 4, 6, 8, 9, 10\}, A \cap B = \{3\}$$

Definition 3.12. The **cardinality** of a set $A$ is the number of elements in the set, and is represented by $n(A)$.

Definition 3.13. A sample space is **discrete** if the elements may be enumerated (it is countable). A **continuous** sample space has infinite cardinality and is uncountable.

3.1.2 Venn Diagrams

3.1.3 The Inclusion/Exclusion Principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$