Final Exam for Math 547, Fall 2011
Due December 6, 1pm

You are allowed to use Hatcher’s book and our class notes, but you should not discuss these problems with anyone or use other sources for your solutions.

1. Let $\Delta^3$ be the standard 3-simplex, and let $K$ be the subcomplex of $\Delta^3$ which is the union of a single 2-simplex with the 1-skeleton of $\Delta^3$. Calculate the fundamental group of $K$, $\pi_1(K,v)$, for a vertex $v \in K$.

2. Let $X = S^2 \cup C$ be the union of the sphere $S^2 = \{(x,y,z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 = 1\}$ with the infinite cylinder $C = \{(x,y,z) \in \mathbb{R}^3 | x^2 + y^2 = 1/4\}$.
   a) Find the homology $H_*(X)$.
   b) Find the local homology $H_*(X,X-p)$ at a point $p \in S^2 \cap C$.

3. a) Describe a CW complex structure for the Klein bottle, $K$, with one 2-cell.
   b) Let $f : K \to S^2$ be the map which collapses $K^1$ (the 1-skeleton) to a point sending $K$ to $K/K^1 = S^2$ and let $r : K \to S^2$ be the map which collapses $K$ to a base point in $S^2$. Show these two maps induce the same map in integral homology.
   c) Use cohomology with coefficients in $\mathbb{Z}/2$ to show these two maps are not homotopic.

4. Let $C$ be the chain complex

   $\cdots \to \mathbb{Z}/9 \xrightarrow{3} \mathbb{Z}/9 \xrightarrow{3} \mathbb{Z}/9 \xrightarrow{3} \mathbb{Z}/9 \to 0$

   where all differentials are multiplication by 3.
   a) Find $H_*(C)$.
   b) Find $\text{Hom}(C, \mathbb{Z}/3)$.
   c) Find $H^*(C; \mathbb{Z}/3)$. 