Problem 1. The following experiment is performed:
An observation is made of a Poisson random variable $N$ with parameter $\lambda$. Then $N$ independent Bernoulli trials are performed, each with probability $p$ of success. Let $Z_i$ be the total number of successes observed in $N$ trials.
(a) Formulate $Z_i$ as a random sum and thereby determine its mean and variance.
(b) Give the formula of the probability mass function of $Z_i$.

Problem 2. A Markov chain $X_0, X_1, \ldots$ on states 0, 1, 2 has the following transition probability matrix:

\[
P = \begin{pmatrix}
0 & 1 & 2 \\
0 & 0.1 & 0.2 & 0.7 \\
1 & 0.9 & 0.1 & 0 \\
2 & 0.1 & 0.8 & 0.1
\end{pmatrix}
\]

and initial distribution $p_0 = P(X_0 = 0) = 0.3$; $p_1 = P(X_0 = 1) = 0.4$, and $p_2 = P(X_0 = 2) = 0.3$. Determine $P(X_i = 0, X_i = 1, X_i = 2)$. 

Problem 3 A Markov chain $X_0, X_1, X_2, \ldots$ has the transition probability matrix

$$
\begin{pmatrix}
0 & 1 & 2 \\
0 & 0.7 & 0.2 & 0.1 \\
1 & 0 & 0.6 & 0.4 \\
2 & 0.5 & 0 & 0.5
\end{pmatrix}
$$

Determine the conditional probability

$$P(X_2 = 1, X_3 = 1 \mid X_1 = 0)$$

Problem 4 Suppose $X_n$ is a two-state Markov chain whose transition probability matrix is

$$
\begin{pmatrix}
0 & 1 \\
\alpha & 1-\alpha \\
1 & \beta \\
1-\beta & \beta
\end{pmatrix}
$$

Then $Z_n = (X_{n-1}, X_n)$ is a Markov chain having the four states $(0, 0), (0, 1), (1, 0)$ and $(1, 1)$. Determine the first row of the transition probability matrix.