Problem 1. Consider a spare parts inventory model in which either 0, 1, or 2 repair parts are demanded in any period, with

\[ P\{X_n = 0\} = 0.4, \quad P\{X_n = 1\} = 0.3, \quad P\{X_n = 2\} = 0.3, \]

and suppose \( s = 0 \) and \( S = 3 \). Determine the transition probability matrix for the Markov chain \( \{X_n\} \), where \( X_n \) is defined to be the quantity on hand at the end of period \( n \).

Problem 2. Find the mean time to reach 3 starting from state 0 for the Markov chain whose transition probability matrix is

\[
P = \begin{pmatrix}
0 & 1 & 2 & 3 \\
0 & 0.4 & 0.3 & 0.2 & 0.1 \\
1 & 0 & 0.7 & 0.2 & 0.1 \\
2 & 0 & 0 & 0.9 & 0.1 \\
3 & 0 & 0 & 0 & 1
\end{pmatrix}
\]
Problem 3: Consider the Markov chain whose transition probability matrix is given by:

\[
P = \begin{pmatrix}
0 & 1 & 2 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(a) Starting in state 1, determine the probability that the Markov chain ends in state 0.

(b) Determine the mean time to absorption.
Problem 4: A coin is tossed repeatedly until two successive heads appear. Find the mean number of tosses required.

Hint: Let $X_n$ be the cumulative number of successive heads. The state space is $0, 1, 2$ and the transition probability matrix is

$$P = \begin{pmatrix}
0 & 1 & 2 \\
1 & \frac{1}{2} & \frac{1}{2} & 0 \\
2 & \frac{1}{2} & 0 & \frac{1}{2}
\end{pmatrix}$$

Determine the mean time to reach state $2$ starting from state $0$ by invoking a first step analysis.