1. A function $f$ satisfies $f''(x) = 3x - \sin x$, $f'(0) = 1$, and $f(0) = 2$. Find $f$.

Solution: The function $f'(x)$ is obtained by integrating $f''(x)$:

$$f'(x) = \int f''(x) \, dx$$
$$f'(x) = \int (3x - \sin x) \, dx$$
$$f'(x) = \frac{3}{2}x^2 + \cos x + C_1$$

The value of $C_1$ is found by using the fact that $f'(0) = 1$.

$$f'(0) = 1$$
$$\frac{3}{2}(0)^2 + \cos 0 + C_1 = 1$$
$$C_1 = 0$$

The function $f(x)$ is obtained by integrating $f'(x)$:

$$f(x) = \int f'(x) \, dx$$
$$f(x) = \int \left( \frac{3}{2}x^2 + \cos x \right) \, dx$$
$$f(x) = \frac{1}{2}x^3 + \sin x + C_2$$

The value of $C_2$ is found by using the fact that $f(0) = 2$.

$$f(0) = 2$$
$$\frac{1}{2}(0)^3 + \sin 0 + C_2 = 2$$
$$C_2 = 2$$

Therefore, the function $f(x)$ is:

$$f(x) = \frac{1}{2}x^3 + \sin x + 2$$
2. Differentiate the function:

\[ F(x) = \int_{\cos x}^{\ln x} \frac{dt}{\cos t + 2} \]

**Solution:** Using the Fundamental Theorem of Calculus Part II and the Chain Rule, the derivative of \( F(x) = \int_{g(x)}^{h(x)} f(t) \, dt \) is:

\[
F'(x) = \frac{d}{dx} \int_{g(x)}^{h(x)} f(t) \, dt \\
= f(h(x)) \cdot \frac{d}{dx} h(x) - f(g(x)) \cdot \frac{d}{dx} g(x)
\]

Applying the formula to the given function \( F(x) \) we get:

\[
F'(x) = \frac{d}{dx} \int_{\cos x}^{\ln x} \frac{dt}{\cos t + 2} \\
= \frac{1}{\cos(\ln x) + 2} \cdot \frac{d}{dx} (\ln x) - \frac{1}{\cos(x) + 2} \cdot \frac{d}{dx} (\cos x) \\
= \frac{1}{\cos(\ln x) + 2} \cdot \left( \frac{1}{x} \right) - \frac{1}{\cos(x) + 2} \cdot (-\sin x)
\]
3. Compute the definite integral: 
\[ \int_{-\pi}^{\pi} x \cos(2x) \, dx \]

**Solution:** We will evaluate the integral using Integration by Parts. Let \( u = x \) and \( v' = \cos(2x) \). Then \( u' = 1 \) and \( v = \frac{1}{2} \sin(2x) \). Using the Integration by Parts formula:

\[ \int_a^b u v' \, dx = [uv]_a^b - \int_a^b u' v \, dx \]

we get:

\[ \int_{-\pi}^{\pi} x \cos(2x) \, dx = \left[ \frac{1}{2} x \sin(2x) \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{1}{2} \sin(2x) \, dx \]

\[ = \left[ \frac{1}{2} x \sin(2x) \right]_{-\pi}^{\pi} - \left[ -\frac{1}{4} \cos(2x) \right]_{-\pi}^{\pi} \]

\[ = \left[ \frac{1}{2} \pi \sin(2\pi) + \frac{1}{4} \cos(2\pi) \right]_{-\pi}^{\pi} \]

\[ = 0 \]
4. Compute the indefinite integrals:

\[
\int (\ln x)^2 \, dx \quad \int \frac{3x - 4}{x^2 - 3x + 2} \, dx \quad \int \frac{(\ln x)^3}{x} \, dx \quad \int x\sqrt{3x + 1} \, dx
\]

Solution:

- The first integral is computed using Integration by Parts. Let \( u = (\ln x)^2 \) and \( v' = 1 \). Then \( u' = \frac{2\ln x}{x} \) and \( v = x \). Using the Integration by Parts formula:

\[
\int uv' \, dx = uv - \int u'v \, dx
\]

we get:

\[
\int (\ln x)^2 \, dx = x(\ln x)^2 - \int \left( \frac{2\ln x}{x} \right) (x) \, dx
\]

\[
= x(\ln x)^2 - 2 \int \ln x \, dx
\]

A second Integration by Parts must be performed. Let \( u = \ln x \) and \( v' = 1 \). Then \( u' = \frac{1}{x} \) and \( v = x \). Using the Integration by Parts formula again we get:

\[
\int (\ln x)^2 \, dx = x(\ln x)^2 - 2 \int \ln x \, dx
\]

\[
= x(\ln x)^2 - 2 \left( x \ln x - \int \left( \frac{1}{x} \right) x \, dx \right)
\]

\[
= x(\ln x)^2 - 2 \left( x \ln x - \int dx \right)
\]

\[
= x(\ln x)^2 - 2x \ln x + 2x + C
\]

- The second integral is computed using Partial Fraction Decomposition. Factoring the denominator and decomposing we get:

\[
\frac{3x - 4}{x^2 - 3x + 2} = \frac{3x - 4}{(x - 1)(x - 2)} = \frac{A}{x - 1} + \frac{B}{x - 2}
\]

Multiplying the equation by \((x - 1)(x - 2)\) we get:

\[
3x - 4 = A(x - 2) + B(x - 1)
\]
Next we plug in two different values of $x$ to get a system of two equations in two unknowns ($A$, $B$). Letting $x = 1$ and $x = 2$ we get:

\[
\begin{align*}
  x = 1 & : \quad 3(1) - 4 = A(1 - 2) + B(1 - 1) \quad \Rightarrow \quad A = 1 \\
  x = 2 & : \quad 3(2) - 4 = A(2 - 2) + B(2 - 1) \quad \Rightarrow \quad B = 2
\end{align*}
\]

Plugging these values of $A$ and $B$ back into the decomposed equation and integrating we get:

\[
\int \frac{3x - 4}{x^2 - 3x + 2} \, dx = \int \left( \frac{1}{x - 1} + \frac{2}{x - 2} \right) \, dx
\]

\[
= \ln|x - 1| + 2 \ln|x - 2| + C
\]

- The third integral is computed using the $u$-substitution method. Let $u = \ln x$. Then $du = \frac{1}{x} \, dx$. We get:

\[
\int \frac{(\ln x)^3}{x} \, dx = \int u^3 \, du
\]

\[
= \frac{1}{4} u^4 + C
\]

\[
= \frac{1}{4} (\ln x)^4 + C
\]

- The fourth integral is computed using the $u$-substitution. Let $u = 3x + 1$. Then $du = 3 \, dx \quad \Rightarrow \quad \frac{1}{3} \, du = dx$ and $x = \frac{1}{3} (u - 1)$ and we get:

\[
\int x \sqrt{3x + 1} \, dx = \int \frac{1}{3} (u - 1) \sqrt{u} \left( \frac{1}{3} \, du \right)
\]

\[
= \frac{1}{9} \int \left( u^{3/2} - u^{1/2} \right) \, du
\]

\[
= \frac{1}{9} \left( \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right) + C
\]

\[
= \frac{2}{45} (3x + 1)^{5/2} - \frac{2}{27} (3x + 1)^{3/2} + C
\]