Math 535: Complex Analysis – Spring 2016 – David Dumas

Practice Midterm Exam

Instructions:
• Complete three of the problems below.
• Each problem is worth 10 points.
• If you complete more than three problems (which is not recommended) your score will be the sum of your three best problem scores.

Problems:
(1) Consider the circles
\[ C_1 = \{ z : |z| = 1 \}, \quad C_2 = \{ z : |z - 1| = 3 \}. \]
Find a linear fractional transformation \( T \) so that \( T(C_1) \) and \( T(C_2) \) are concentric circles.

(2) Find a conformal mapping from the open first quadrant to the complement of the closed unit disk, i.e. from
\[ \{ z : 0 < \arg(z) < \frac{\pi}{2} \} \]
to
\[ \{ z : |z| > 1 \}. \]

(3) Let \( \gamma \) be the circle \( |z| = 535 \) oriented counter-clockwise. Calculate:
\[ \int_{\gamma} \frac{\cos(z/2)}{4z^2 - \pi^2} \, dz \]

(4) Suppose \( f \) is a holomorphic function with a zero of order 2 at \( z = 0 \). Show that there is an analytic function \( g \) on an open disk centered at \( z = 0 \) such that \( f(z) = g(z)^2 \) on their common domain.

(5) Let \( f \) be an entire function, and suppose that \( f \) does not take on any positive real values. Show that \( f \) is constant.