1. (20 pts) Let $Q_k$ be the $k$-hypercube. Assume that $Q_k$ has a Hamiltonian cycle. Show that for each $2 \leq p \leq k$, $Q_k$ has a cycle of length $2^p$.

2. Let $D = (V, A)$ be a directed graph.
   (a) (5 pts) Give the definition of: $D$ is strongly connected.
   (b) (15 pts) Assume that $D$ is strongly connected, has no loops, and between any two distinct vertices $u, v$ there is at most one arc, either $(u, v)$ or $(v, u)$ in $A$, but not both. (It is possible that there no arc between $u$ and $v$.) Show that each $v$ lies on a directed cycle of length 3 at least.

3. (20 pts.) Let $T = (G, E)$ be a tree. Show that if $T$ has a vertex of degree $k \geq 2$, then it has at least $k$ leaves.

4. (20 pts.) Let $G = (V, E)$ be a bipartite graph with the partition of $V$ to $X$ and $Y$. Assume that every subset $S \subseteq X$ satisfies
   \[ |S| \leq |N(S)| + 1. \]
   Show that $G$ has a matching that saturates every vertex of $X$ with the exception of at most one vertex.

5. Let $G = (V, E)$ be an undirected graph. Denote $n(G) := |V|$.
   (a) (6 pts.) Define: independent set, maximum independent set (its size $\alpha(G)$), edge cover, minimum edge cover (its size $\beta'(G)$).
   (b) (14 pts.) Show $\beta'(G) \geq \alpha(G)$ if $G$ has no isolated vertices.