Problem 1.

1. Find the determinant of $A$: 

$$A = \begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 11 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix}$$

2. Let 

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & -3 & s \end{bmatrix}$$

   (a) Find the adjoint (adjugate) matrix of $A$.
   
   (b) For which values of $s$ is $A$ not invertible.
   
   (c) Find the inverse of $A$ using the adjoint matrix for those values of $s$ for which $A$ is invertible.

3. Let 

$$A = \begin{bmatrix} 3 & 2 \\ 5 & 0 \end{bmatrix}$$

   Consider a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x) = Ax$.

   What is the area of the image of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2}$? (Recall that the area of this ellipsoid is $\pi ab$.)

Problem 2.

1. Find if the following sets are subspaces in a given vector space:

   (a) All vectors $x = (x_1, x_2, x_3)^T$ satisfying $x_1 x_2 x_3 = 0$.
   
   (b) All polynomial of degree $n$ at most satisfying the condition $p(-1) = 0$.

2. Let $v_1, v_2, v_3, v_4$ be the following vectors in $\mathbb{R}^4$:

   $$v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 3 \\ -1 \\ -5 \\ 1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$ 

   Is $v_1, v_2, v_3, v_4$ a basis in $\mathbb{R}^4$? If not find a basis in span($v_1, v_2, v_3, v_4$).

Problem 3. Let 

$$A = \begin{bmatrix} -1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$$

1. The rank of $A$.

2. The nullity of $A$ (the dimension of the null space of $A$).

3. The rank of $A^T$.

4. The nullity of $A^T$.

5. A basis in the column space of $A$.

6. A basis in the row space of $A$. 

7. A basis in the null space of $A$.
8. A basis in the row space of $A^\top$.
9. A basis in the column space of $A^\top$.
10. A basis in the null space of $A^\top$.

**Problem 4.** Let $P_2$ be the vector space of all polynomials of degree at most two. Let $B = \{1, t, t^2\}$ be the standard basis in $P_2$.

1. Let $T : P_2 \to P_2$ be the linear transformation: $T(f) = -f'$, where $f'$ is the derivative of $f$. Find the matrix of $T$ relative to $B$.
2. Let $C = \{1 + t, t + t^2, 1 - t - t^2\}$.
   (a) Find change-of-coordinate matrix from $C$ to $B$ ($P_{B\leftarrow C}$).
   (b) Find the coordinates of $1 + t + t^2$ in basis $C$.

**Problem 5.** Let $A = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$.

1. Is $A$ a stochastic matrix?
2. Diagonalize $A$.
3. Find $\lim_{k \to \infty} A^k$