1. (2 pts) State briefly what does it mean that $b$ is a linear combination of $v_1, v_2, v_3$?

2. Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 9 \\ 9 \end{bmatrix}.$$ 

(a) (6 pts) Is $b$ a linear combination of $v_1, v_2, v_3$?

(b) (2 pts) Do $v_1, v_2, v_3$ span $\mathbb{R}^3$? (Justify briefly.)

1. \(x_1 v_1 + x_2 v_2 + x_3 v_3 = b\)

2. \(x_1 + 2x_2 - 3x_3 = 0, \quad 2x_1 + 3x_2 + 4x_3 = 9, \quad 4x_1 + 7x_2 - 2x_3 = 9\)

\[
\begin{bmatrix}
1 & 2 & -3 & 0 \\
0 & -1 & 10 & 9 \\
0 & -1 & 10 & 9
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & -3 & 0 \\
0 & 1 & 10 & 9 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Hence $b$ is a linear combination of $v_1, v_2, v_3$.

(b) The REF of the coefficient matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 4 \\ -2 & -2 & -2 \end{bmatrix}$ has 2 pivots. Hence there is no $x$ for which $Ax = b$.

Thus $v_1, v_2, v_3$ do not span $\mathbb{R}^3$. 

\[\text{No pivot in the last column - the system is solvable}\]