Show all work. Unjustified answer yields no credit.

Assume that $V$ is IPS over $C$.

1. (2 pts) Is it true that $\|x + y\|^2 = \|x\|^2 + \|y\|^2 + 2\langle x, y \rangle$? If yes, prove it. If no state the correct formula and prove it.

2. (a) (5 pts) Prove the triangle inequality $\|x + y\| \leq \|x\| + \|y\|$.
   (b) (3 pts) State and prove the exact condition of equality in the triangle inequality.

1. No. $\|x + y\|^2 = \langle x + y, x + y \rangle = \langle x, x \rangle + \langle y, y \rangle + 2\Re \langle x, y \rangle$

2. $|\langle x, x \rangle| \leq \|x\| \|x\|, \quad |\langle x, y \rangle| \leq \|x\| \|y\|$
   C-S Inequality. So
   $\|x + y\|^2 \leq \|x\|^2 + \|y\|^2 + 2\|x\| \|y\| \leq (\|x\| + \|y\|)^2$

3. Equality in the triangle inequality. If $x = 0$ or $y = 0$.
   OK. So assume that $x \neq 0, y \neq 0$.
   To have equality in C-S inequality, we must have
   $X, Y \in C, \text{deg.}, \text{i.e.,} \quad Y = \alpha X, \text{at } C$
   $\|x + y\| = \|x + \alpha x\| = \| (1 + \alpha) x \| = |1 + \alpha| \|x\|$
   $\|x + y\| = \|x\| + |\alpha| \|x\| = \|x\| + |\alpha| \|x\| = (1 + |\alpha|) \|x\|$
   So we must have $|1 + \alpha| = 1 + |\alpha| \implies |\alpha| > 0$.
   This will happen off $\alpha > 0$. So
   $Y = \alpha X, \quad \alpha > 0$.