Show all work. Unjustified answer yields no credit.

1. (6 pts) Find the best least square fit to \( y = ax + b \) to the following four points in the plane \((x, y) (-1, 1), (0, 1), (1, 2), (2, -1)\).

2. (4 pts) Let \( z_1, \ldots, z_n \) be \( n \) distinct points in the complex plane \( \mathbb{C} \). Let \( P_n \) be the vector space of polynomials in \( z \in \mathbb{C} \) with complex coefficients of degree \( n \) at most. For \( f(z), g(z) \in P_n \) define

\[
(f, g) = \sum_{j=1}^{n} f(z_j) \overline{g(z_j)}.
\]

Is \((f, g)\) inner product on \( P_n \)?

\[A^TA = \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \]

\[A^TA = \frac{1}{20} \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix} \]

\[y = bx + a = -\frac{1}{2} x + 1\]

b. Take \( p(z) = (z-z_1)(z-z_2) \neq 0 \) \( p_01 \).

But \( \langle p(z), p(z) \rangle = \sum_{j=1}^{n} p(z_j) \overline{p(z_j)} = 0 \)

So not inner product.