Show all work. Unjustified answer yields no credit. You can use any part of this problem, without showing it, to show another part of this problem. In the digraph below above each edge $e$ there are two numbers: $(c, f)$ satisfying $c(e) \geq f(e) \geq 0$. Undergraduates: Do parts 1-3. Graduates 1 - 4.

1. (ug: 2 pts, gr: 1 pt gr.) Show that $f$ is an $(s, t)$ flow.

2. (ug: 5 pts, gr: 4pts) Find a maximum $(s, t)$ flow.

3. (ug: 3 pts, gr: 3 pts.) Find a minimum capacity cut.

4. (graduates only 2 pts) Given a digraph $G = (V, E)$ with positive capacities $c : E \to (0, \infty)$. Assume that not all capacities are integers. How would you find a maximal integer flow?

\[ f(s_{r}) = 1+1-(1+1)=0 \]
\[ f(q) = 2-(1+1)=0 \]
\[ f(r) = 1+1-(1+1)=0 \]
\[ f(b) = 1+1-2=0 \]

2. $G(f)$ digraph with $E=(E)$

\[ \text{The only } s-t \text{ path in } G(f) \text{ is} \]
\[ S\rightarrow q \rightarrow a \rightarrow p \rightarrow b \rightarrow t \]
\[ \text{On } ap \text{ is } 1 \text{ so we can augment by } 1 \]

\[ \text{The new flow} \]
\[ f_{1} \]
\[ \text{IN } G(f_{1}) \text{ } s \text{ is connected to } q, \text{ } q \text{ to } a \text{ and } s \text{ is not connected to anything } \text{ from } a. \]

\[ \text{a. } \text{So no augmenting path from } s \text{ to } t. \text{ Hence } f_{1} \text{ is the maximal} \]

\[ \text{3. } (s,t) \text{ minimal capacity cut - all points including } s \text{ that} \]
\[ \text{can be reached from } s: R = \{s, q, a\}. \text{ Its capacity} \]
\[ c(s_{p}) + c(q_{b}) + c(q_{p}) + c(a_{t}) = 1+1+1=3. \]

\[ \text{4. Replace } c(uw) \text{ by } \lceil c(uw) \rceil \text{ - the biggest integer not bigger} \]
\[ \text{then } c(uw). \text{ Now solve the maximum flow problem.} \]