Most of the problems are from [1] unless stated otherwise.

1. Problem 2.2.7, page 92. Let $K_n$ be a complete graph on $n$ vertices. By symmetry, each edge appears in $k(n)$ of spanning trees in $K_n$. Let us count the number of edges appearing in all spanning trees of $K_n$ in two ways. First, by Cayley’s theorem there are $n^{n-2}$ spanning trees. Each tree has $n-2$ edges. Hence the total number of edges in $n^{n-2}$ spanning trees is $(n-1)^n$. On the other hand there are $n(n-1)$ edges in $K_n$. Each one appears in $k(n)$ spanning trees. Hence the total number of edges in all spanning trees is $n(n-1)k(n)$. Thus $(n-1)^n = n(n-1)k(n)$. Therefore $k(n) = 2^{n-3}$. Finally the number of spanning trees in $K_n - e$ is $n^{n-2} - 2n^{n-3} = n^{n-3}(n-2)$.

2. Problem 2.2.10, page 92.

(a) $m = 1$. $K_{2,1}$ is a path on 3 vertices. It has a unique spanning tree $K_{2,1}$.

(b) $m \geq 2$. Let $T$ be a spanning tree of $K_{2,m} = (V,E)$, where $X \cup Y$, $X = \{u,v\}$ and $Y = [m] := \{1, \ldots, m\}$. It will have $m+1$ edges. So $u$ has to be connected to $p \geq 1$ vertices in $Y$ and $v$ is connected to $q \geq 1$ vertices in $Y$ and $p+q = m+1$. Hence $u$ and $v$ has exactly one common neighbor $w \in Y$. Vice versa, choose $w \in Y$ and connected it to $u$ and $v$. Let $Y' := Y - \{w\}$. Divide $Y'$ to a union of two disjoint sets $Y_1 \cup Y_2$, where $|Y_1| = p - 1, |Y_2| = q - 1$. (Note that $Y_i$ may be empty.) Then connect all the vertices of $Y_1$ to $u$ and all the vertices of $Y_2$ to $v$. This is a spanning tree.

The number of spanning trees is. The choice of $w$: $m$ choices. All subsets $Y_1$ of $Y'$ is $2^{m-1}$. (Each element in $Y'$ has two choices to be or not to be in $Y_1$.) Hence the number of spanning trees is $m2^{m-1}$. (Still true for $m = 1$)

The number of nonisomorphic trees depends on the value of $p$. By switching $u$ and $v$ we can assume that $p \leq \lceil \frac{m+1}{2} \rceil$. (Recall $p + q = m + 1$!) So the number of nonisomorphic trees is $\lceil \frac{m+1}{2} \rceil$. 

References


   http://www.ecp6.jussieu.fr/pageperso/bondy/books/gtwa/gtwa.html

   http://lib.myilibrary.com.proxy.cc.uic.edu/Open.aspx?id=134028&loc=&srch=undefined&src=0